The Multiple Access Channel with Common Rate-Limited Feedback

Dor Shaviv and Yossef Steinberg

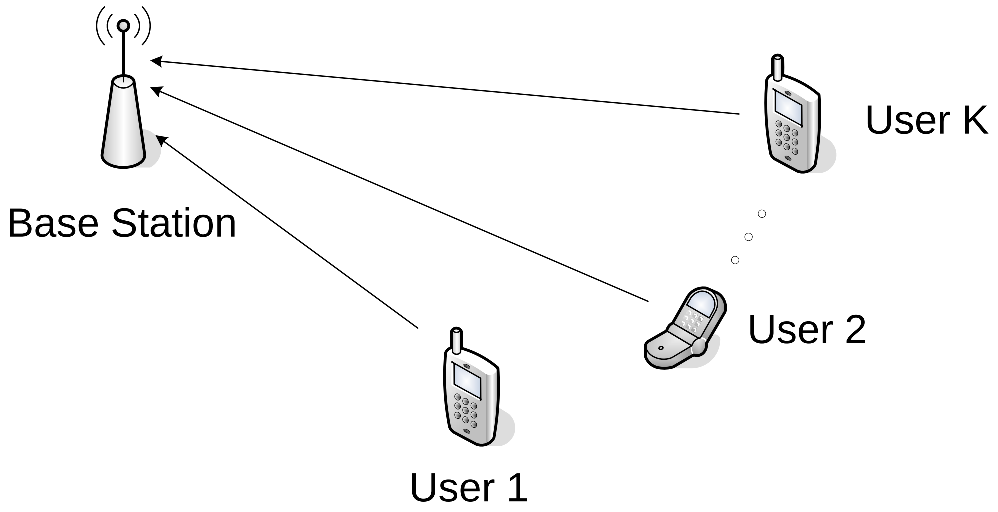
Technion—Israel Institute of Technology

ysteinbe@ee.technion.ac.il

Introduction

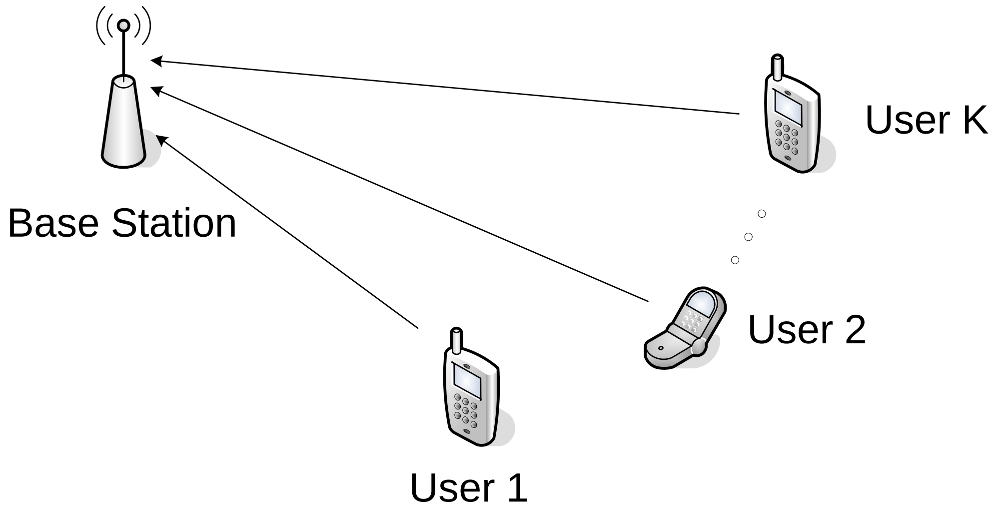
Motivation

Uplink in a Cellular Communication System



Motivation

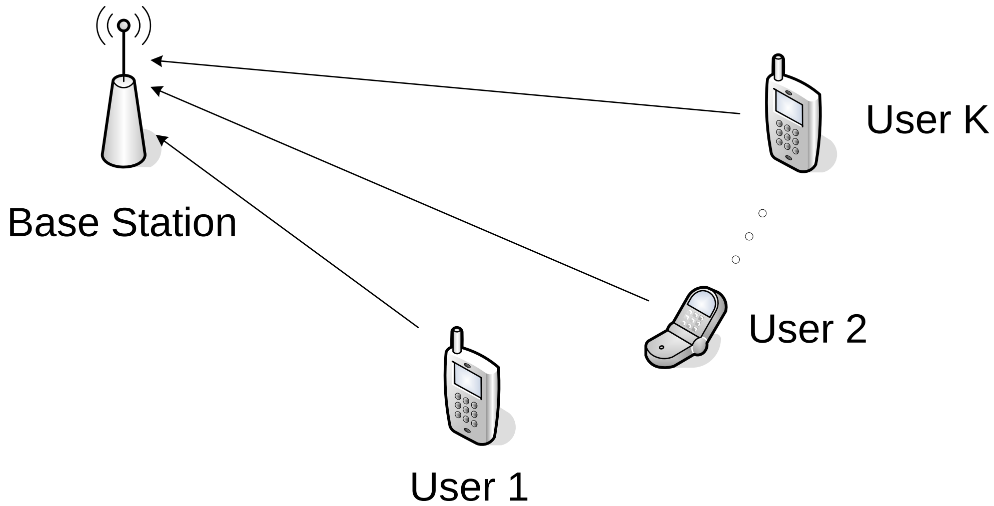
Uplink in a Cellular Communication System



- Channel depends on state (fading, multipath, interference, etc).

Motivation

Uplink in a Cellular Communication System

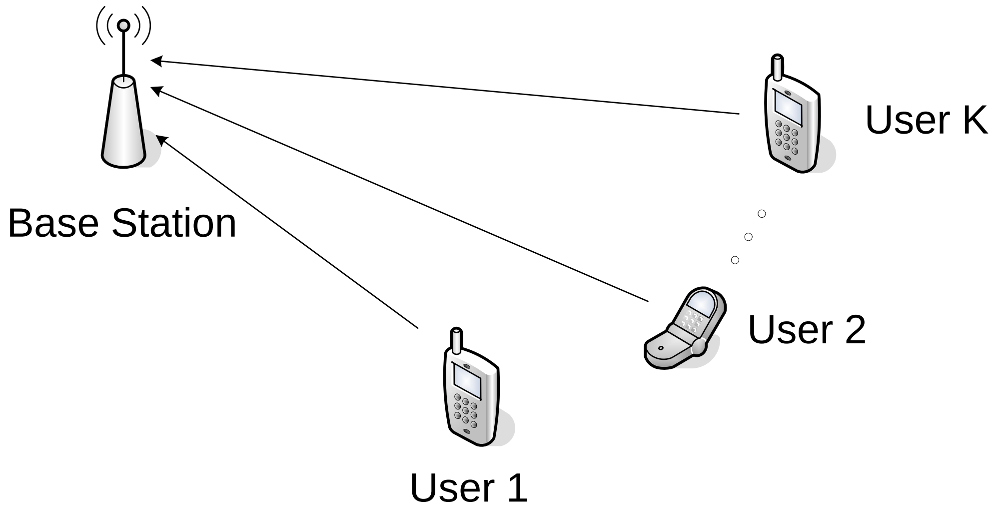


- Channel depends on state (fading, multipath, interference, etc).
- The state cannot be measured by the users. *Reflected* at the channel output (the signal available at the base station):

$$Y_i = \sum_{k=1}^{K} S_{k,i} X_{k,i} + \nu_i , \quad i = 1, 2, \ldots$$

Motivation

Uplink in a Cellular Communication System



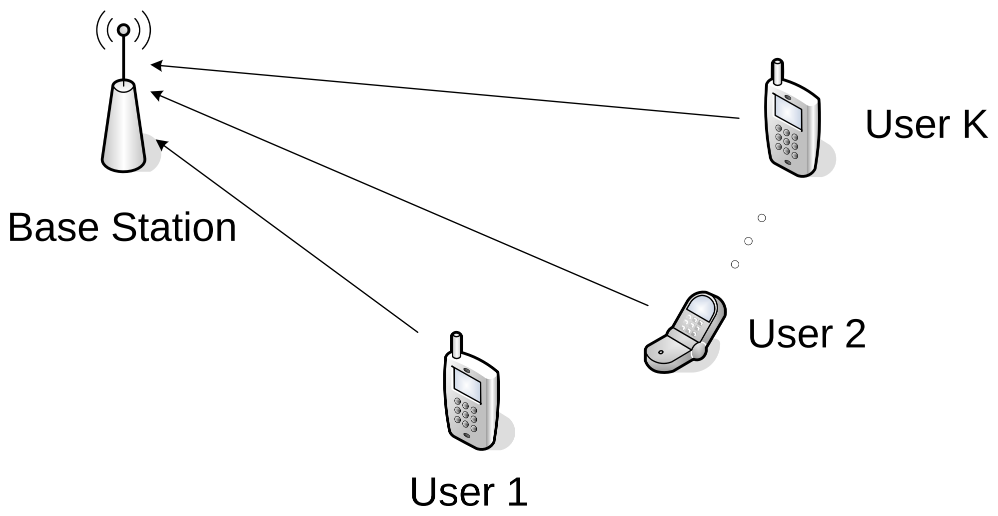
- Channel depends on state (fading, multipath, interference, etc).
- The state cannot be measured by the users. *Reflected* at the channel output (the signal available at the base station):

$$Y_i = \sum_{k=1}^{K} S_{k,i} X_{k,i} + \nu_i, \quad i = 1, 2, \ldots$$

- Users cannot directly cooperate, or exchange information.

Motivation

Uplink in a Cellular Communication System



- Channel depends on state (fading, multipath, interference, etc).
- The state cannot be measured by the users. *Reflected* at the channel output (the signal available at the base station):

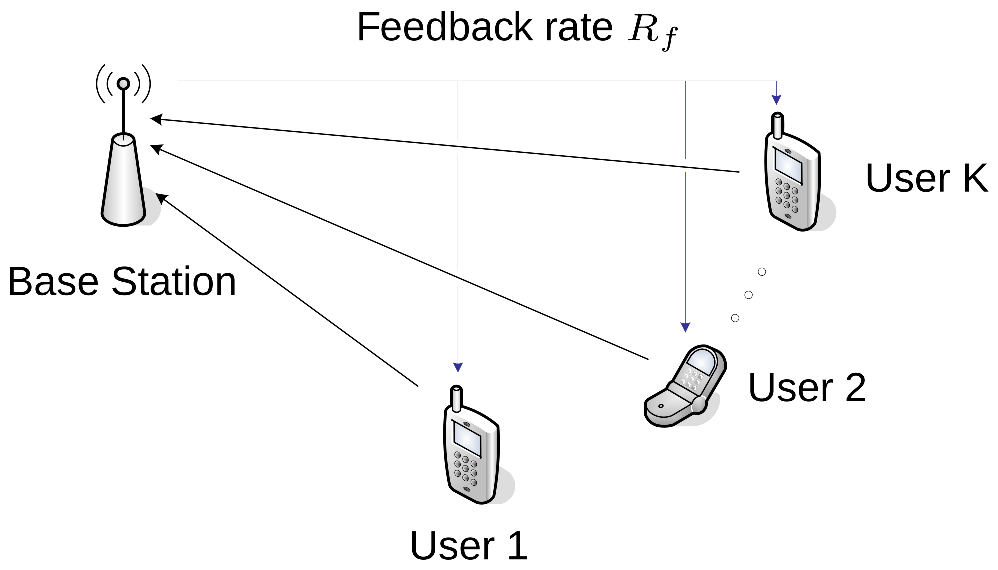
$$Y_i = \sum_{k=1}^{K} S_{k,i} X_{k,i} + \nu_i, \quad i = 1, 2, \ldots$$

- Users cannot directly cooperate, or exchange information.

$\Longrightarrow$ Use feedback from base station to the users

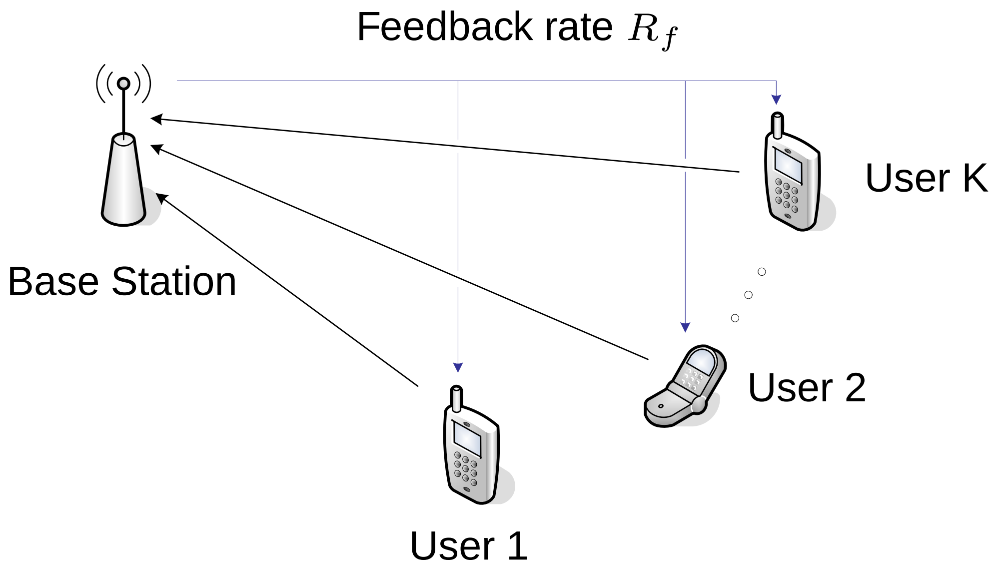
...Motivation

Uplink in a Cellular Communication System



...Motivation

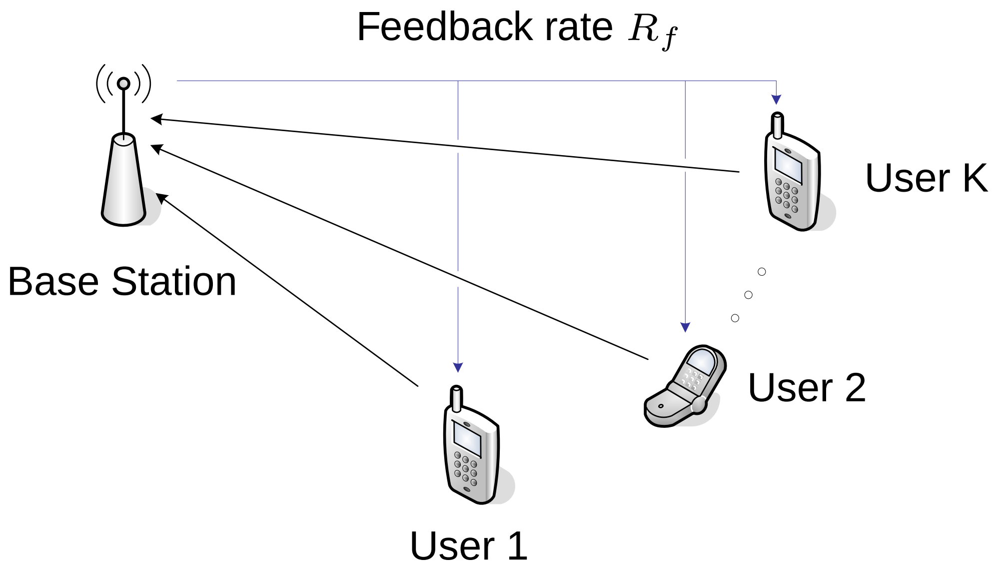
Uplink in a Cellular Communication System



In a cellular system, feedback resources are expensive. Therefore:

...Motivation

Uplink in a Cellular Communication System

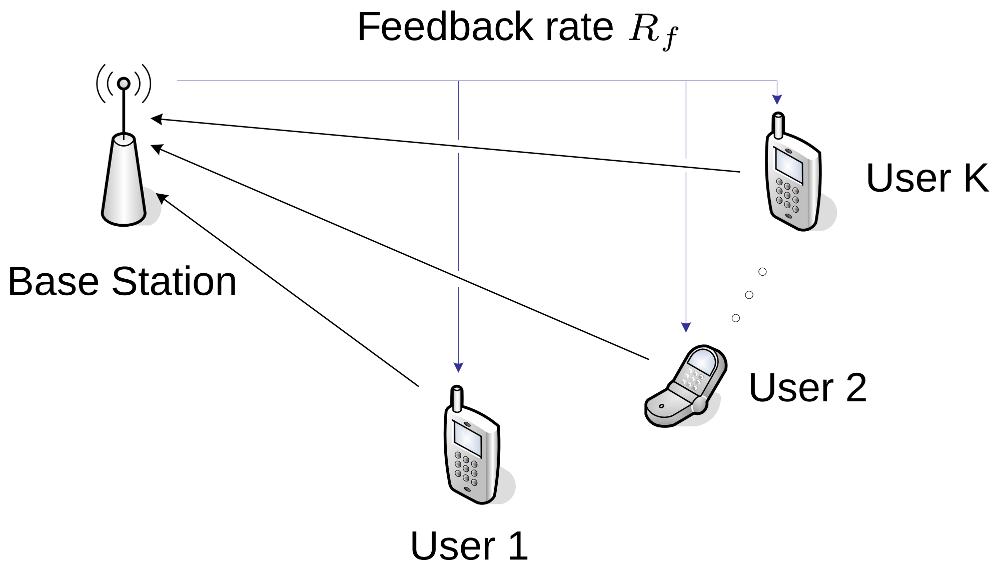


In a cellular system, feedback resources are expensive. Therefore:

- Rate limited feedback, R_f

...Motivation

Uplink in a Cellular Communication System

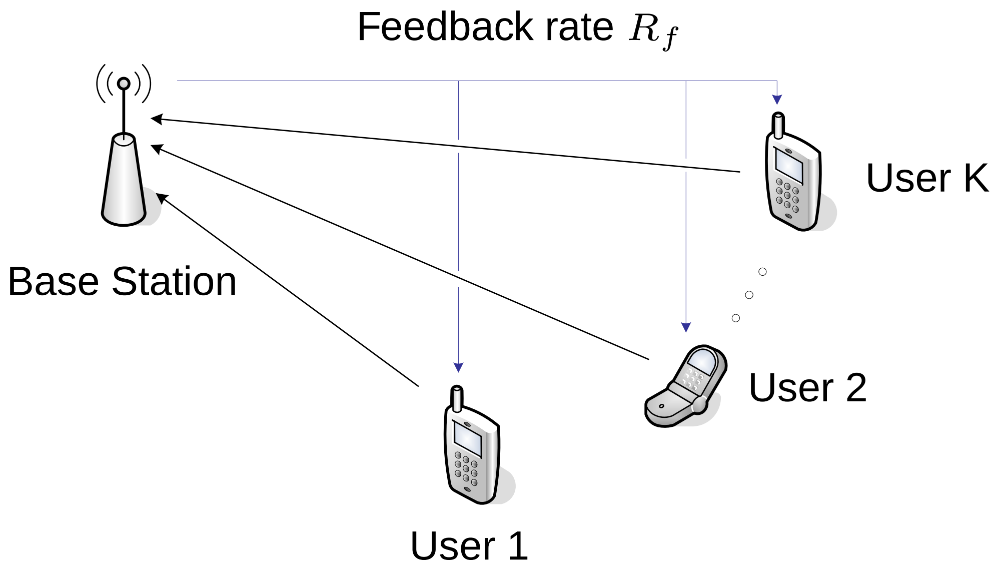


In a cellular system, feedback resources are expensive. Therefore:

- Rate limited feedback, R_f
- Common feedback link: all users get the same stream

...Motivation

Uplink in a Cellular Communication System



In a cellular system, feedback resources are expensive. Therefore:

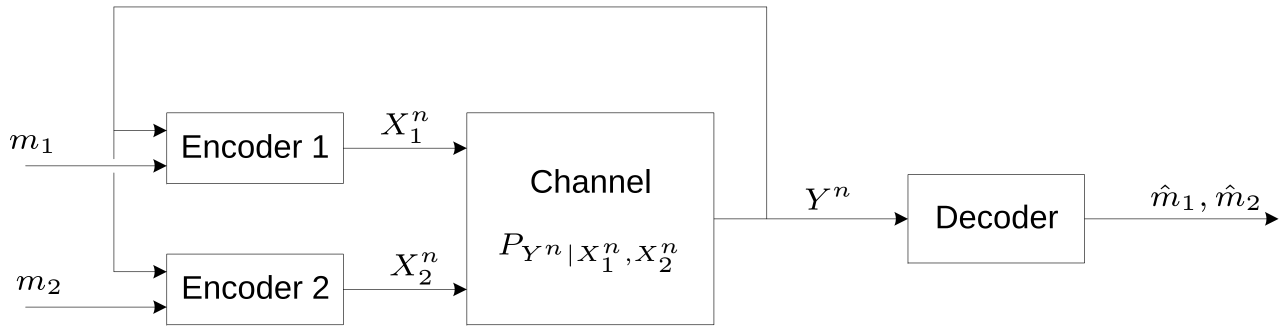
- Rate limited feedback, R_f
- Common feedback link: all users get the same stream
- Portions of the feedback information may be dedicated

Outline

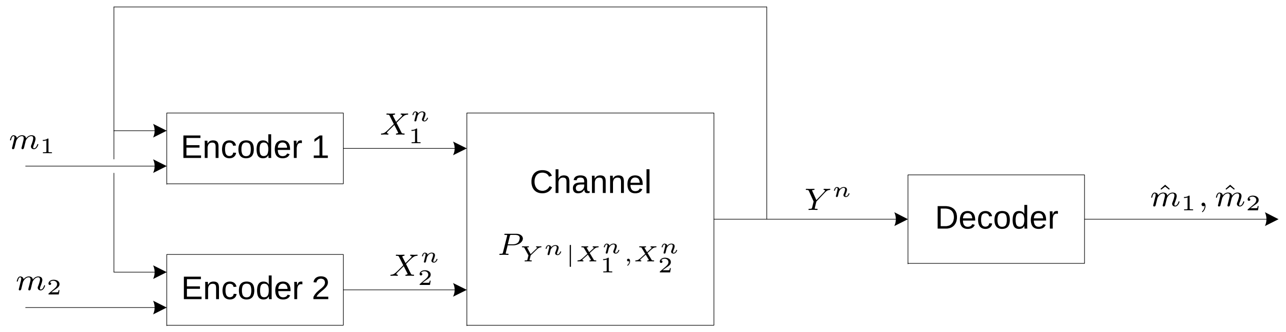
- Channel and feedback models
  - Perfect feedback
  - Generalized (noisy) feedback
  - Rate-limited feedback
- Main result
- The coding scheme

Channel and feedback models

The memoryless MAC with perfect feedback



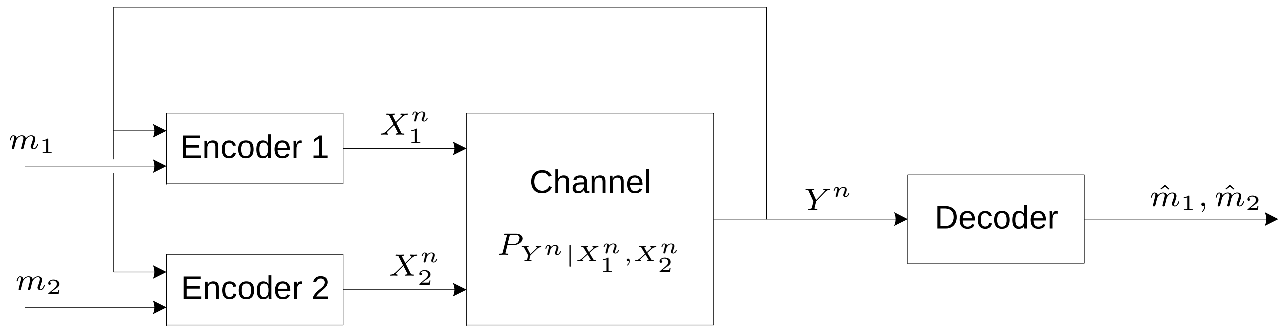
The memoryless MAC with perfect feedback



- The channel is memoryless

$$P_{Y^n|X_1^n,X_2^n}(y^n|x_1^n, x_2^n) = \prod_{i=1}^{n} P_{Y|X_1,X_2}(y_i|x_{1,i}, x_{2,i})$$

The memoryless MAC with perfect feedback



- The channel is memoryless

$$P_{Y^n|X_1^n,X_2^n}(y^n|x_1^n,x_2^n) = \prod_{i=1}^{n} P_{Y|X_1,X_2}(y_i|x_{1,i},x_{2,i})$$

- Channel encoders:

$$X_{k,i} = f_k(m_k, y_1^{i-1}), \quad k = 1, 2.$$

MAC with perfect feedback *(cont'd)*

Previous work

MAC with perfect feedback *(cont'd)*

Previous work

- Gaarder and Wolf 1975 – observed that feedback can increase the capacity region of memoryless MAC. The role of feedback is to establish cooperation between the users.

MAC with perfect feedback (cont'd)

Previous work

- Gaarder and Wolf 1975 – observed that feedback can increase the capacity region of memoryless MAC. The role of feedback is to establish cooperation between the users.
- Cover and Leung 1981 – derived achievable region. Introduced superposition of information and block Markov coding, in the context of MAC.

MAC with perfect feedback *(cont'd)*

Previous work

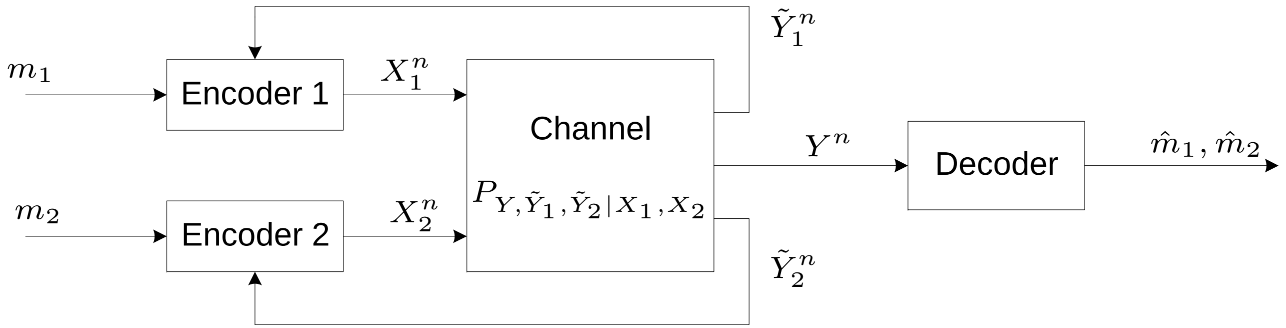
- Gaarder and Wolf 1975 – observed that feedback can increase the capacity region of memoryless MAC. The role of feedback is to establish cooperation between the users.
- Cover and Leung 1981 – derived achievable region. Introduced superposition of information and block Markov coding, in the context of MAC.
- Bross and Lapidoth 2005 – derived a new inner bound that contains the Cover-Leung region as a special case. Showed that can be strictly better than the Cover-Leung region for some channels.

MAC with perfect feedback *(cont'd)*

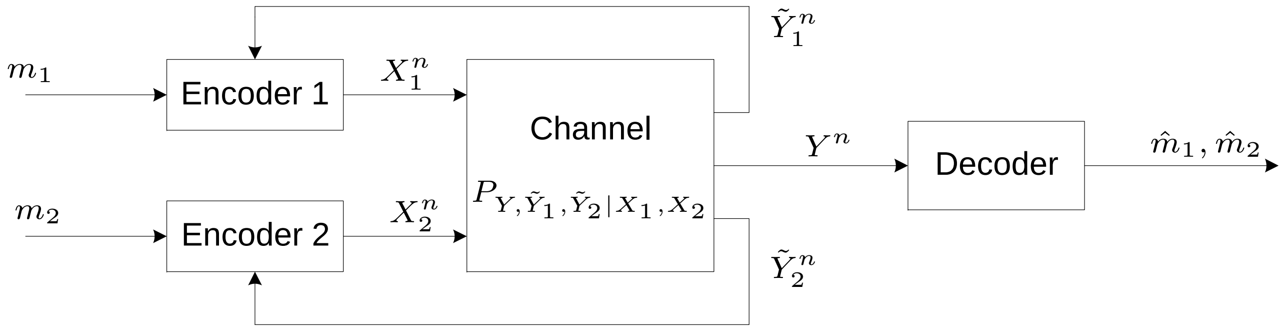
Previous work

- Gaarder and Wolf 1975 – observed that feedback can increase the capacity region of memoryless MAC. The role of feedback is to establish cooperation between the users.
- Cover and Leung 1981 – derived achievable region. Introduced superposition of information and block Markov coding, in the context of MAC.
- Bross and Lapidoth 2005 – derived a new inner bound that contains the Cover-Leung region as a special case. Showed that can be strictly better than the Cover-Leung region for some channels.
- Ozarow 1984 – derived the capacity region of Gaussian two user MAC with perfect feedback. Used a deterministic coding scheme based on the Schalkwijk-Kailath algorithm. Ozarow's region is strictly better than the Cover-Leung region for the Gaussian MAC.

MAC with generalized feedback

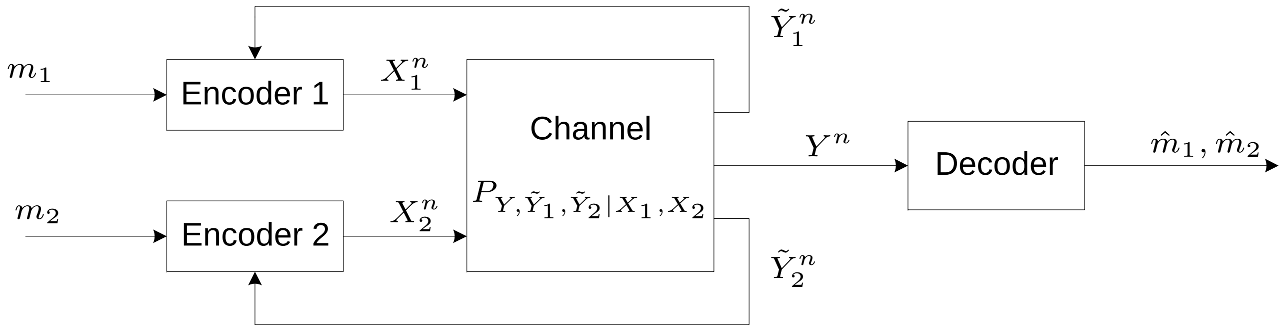


MAC with generalized feedback



- $\tilde{Y}_1$, $\tilde{Y}_2$ can be viewed as noisy versions of the channel output Y.

MAC with generalized feedback



- $\tilde{Y}_1$, $\tilde{Y}_2$ can be viewed as noisy versions of the channel output Y.
- Channel encoders:

$$X_{k,i} = f_k(m_k, \tilde{y}_{k,1}^{i-1}), \quad k = 1, 2.$$

MAC with generalized feedback (cont'd)

Previous work

MAC with generalized feedback *(cont'd)*

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.

MAC with generalized feedback (cont'd)

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:

MAC with generalized feedback *(cont'd)*

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:
  - Strictly better than $\mathcal{C}_{\text{MAC}}$ without feedback, $\forall$ feedback noise power

MAC with generalized feedback *(cont'd)*

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:
  - Strictly better than $\mathcal{C}_{\text{MAC}}$ without feedback, $\forall$ feedback noise power
  - For feedback noise power $\to 0$, sum rate coincides with Ozarow's sum rate.

MAC with generalized feedback *(cont'd)*

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:
  - Strictly better than $\mathcal{C}_{\text{MAC}}$ without feedback, $\forall$ feedback noise power
  - For feedback noise power $\to 0$, sum rate coincides with Ozarow's sum rate.

    $\Longrightarrow$ At the limit of *almost perfect feedback*, Lapidoth-Wigger scheme provides optimal sum capacity.

MAC with generalized feedback (cont'd)

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:
  - Strictly better than $\mathcal{C}_{\text{MAC}}$ without feedback, $\forall$ feedback noise power
  - For feedback noise power $\rightarrow 0$, sum rate coincides with Ozarow's sum rate.

    $\Longrightarrow$ At the limit of *almost perfect feedback*, Lapidoth-Wigger scheme provides optimal sum capacity.
  - Smooth transition from deterministic to random coding schemes.

MAC with generalized feedback *(cont'd)*

Previous work

- Carleial 1982 – suggested the model, and derived an achievable region.
- Lapidoth and Wigger 2006 – studied Gaussian MAC with noisy feedback.

  Suggested achievable region $\mathcal{R}_i^{(LW)}$ with the properties:
  - Strictly better than $\mathcal{C}_{\text{MAC}}$ without feedback, $\forall$ feedback noise power
  - For feedback noise power $\to 0$, sum rate coincides with Ozarow's sum rate.

    $\Longrightarrow$ At the limit of *almost perfect feedback*, Lapidoth-Wigger scheme provides optimal sum capacity.
  - Smooth transition from deterministic to random coding schemes.
- Gastpar and Kramer 2006 – Studied Gaussian MAC with noisy feedback. Outer bounds based on dependence-balance technique. Achievability results obtained via block coding and linear feedback.

MAC with generalized feedback (cont'd)

The model of generalized (noisy) feedback is more realistic than noiseless feedback. But it still suffers from a few shortcomings

MAC with generalized feedback (cont'd)

The model of generalized (noisy) feedback is more realistic than noiseless feedback. But it still suffers from a few shortcomings

- The *rate of use* of the feedback link is the same as that of the main channel.

MAC with generalized feedback (cont'd)

The model of generalized (noisy) feedback is more realistic than noiseless feedback. But it still suffers from a few shortcomings

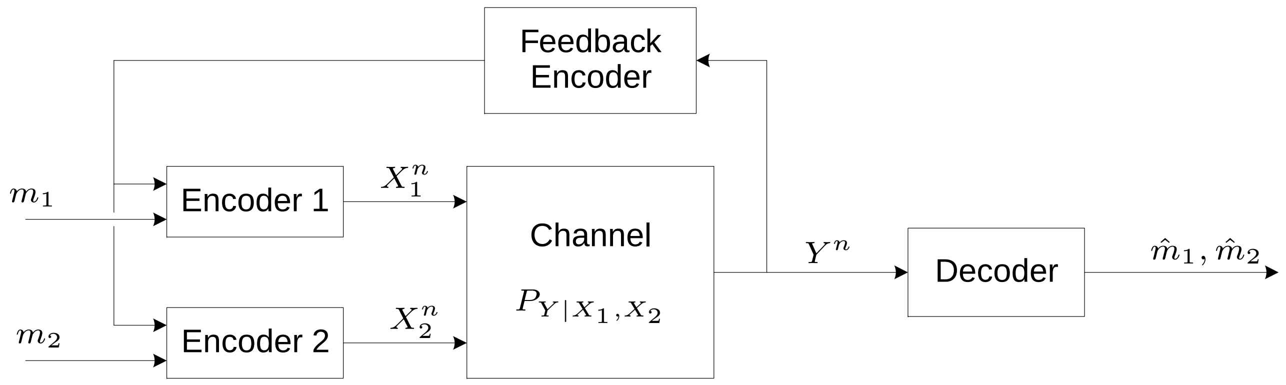
- The *rate of use* of the feedback link is the same as that of the main channel.
- Does not take into account processing (coding) at the input to the feedback link.

MAC with generalized feedback *(cont'd)*

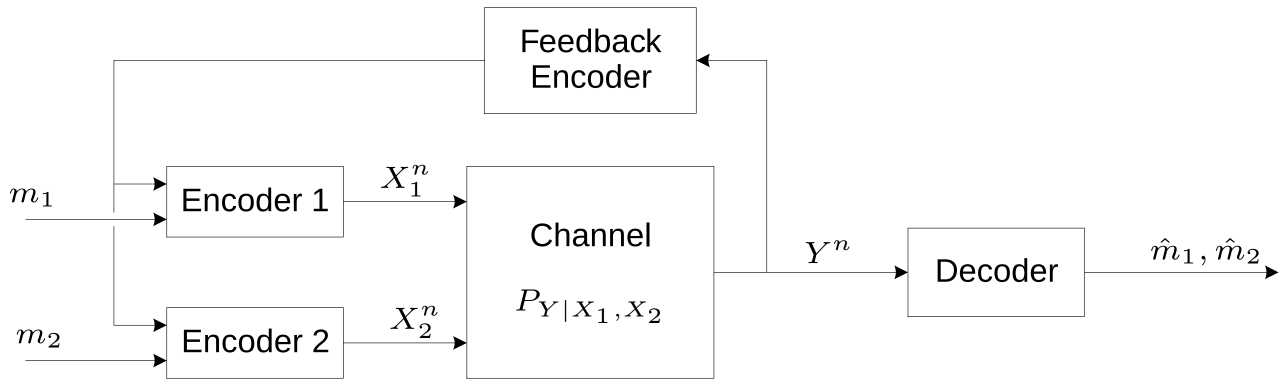
The model of generalized (noisy) feedback is more realistic than noiseless feedback. But it still suffers from a few shortcomings

- The *rate of use* of the feedback link is the same as that of the main channel.
- Does not take into account processing (coding) at the input to the feedback link.
- Hard to compare gains vs. cost of feedback.

MAC with rate-limited feedback



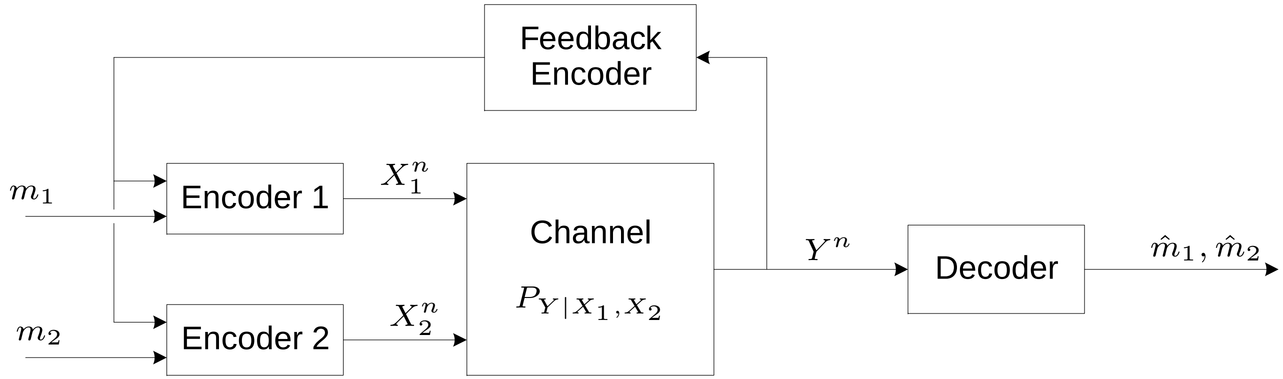
MAC with rate-limited feedback



- Channel encoders:

$$X_{k,i} = f_k(m_k, f_0(y_1^{i-1})), \quad k = 1, 2.$$

MAC with rate-limited feedback

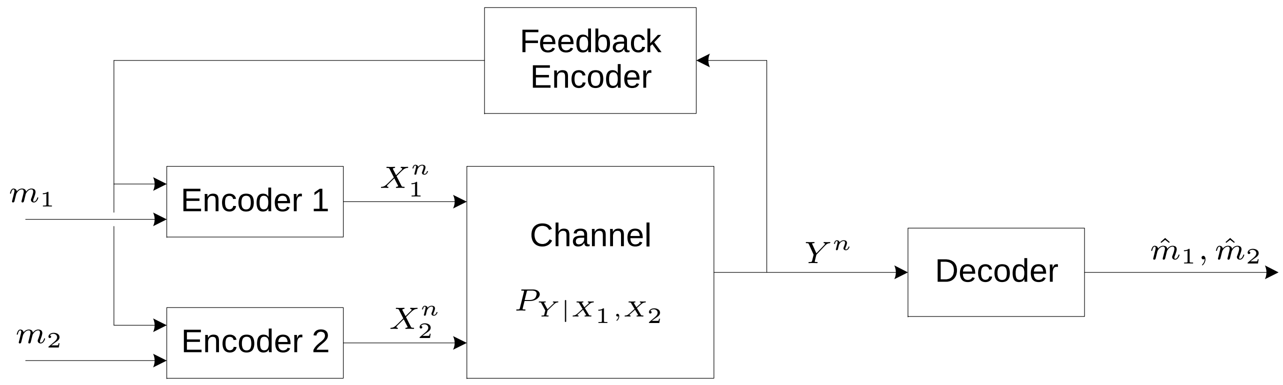


- Channel encoders:

$$X_{k,i} = f_k(m_k, f_0(y_1^{i-1})), \quad k = 1, 2.$$

How feedback rate is measured?

MAC with rate-limited feedback



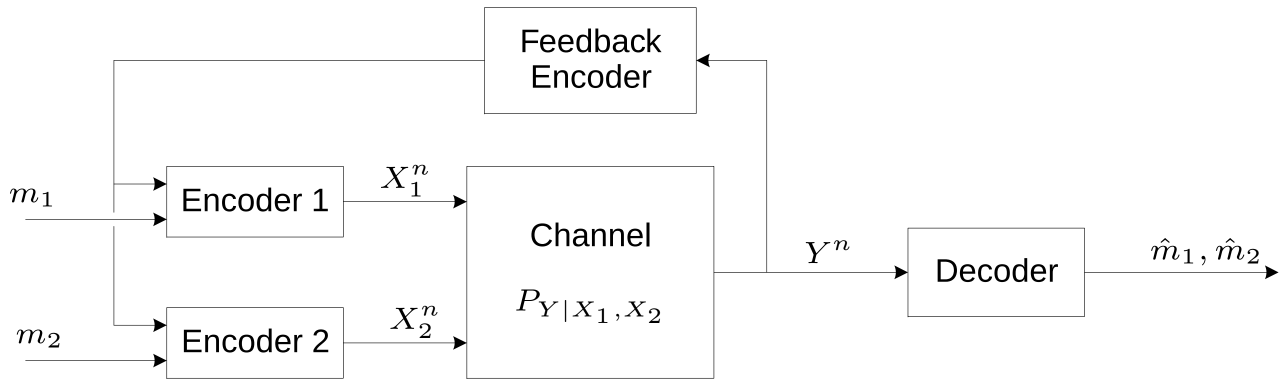
- Channel encoders:

$$X_{k,i} = f_k(m_k, f_0(y_1^{i-1})), \quad k = 1, 2.$$

How feedback rate is measured?

1. Should be more general than just one letter quantization

MAC with rate-limited feedback



- Channel encoders:

$$X_{k,i} = f_k(m_k, f_0(y_1^{i-1})), \quad k = 1, 2.$$

How feedback rate is measured?

1. Should be more general than just one letter quantization
2. Causal

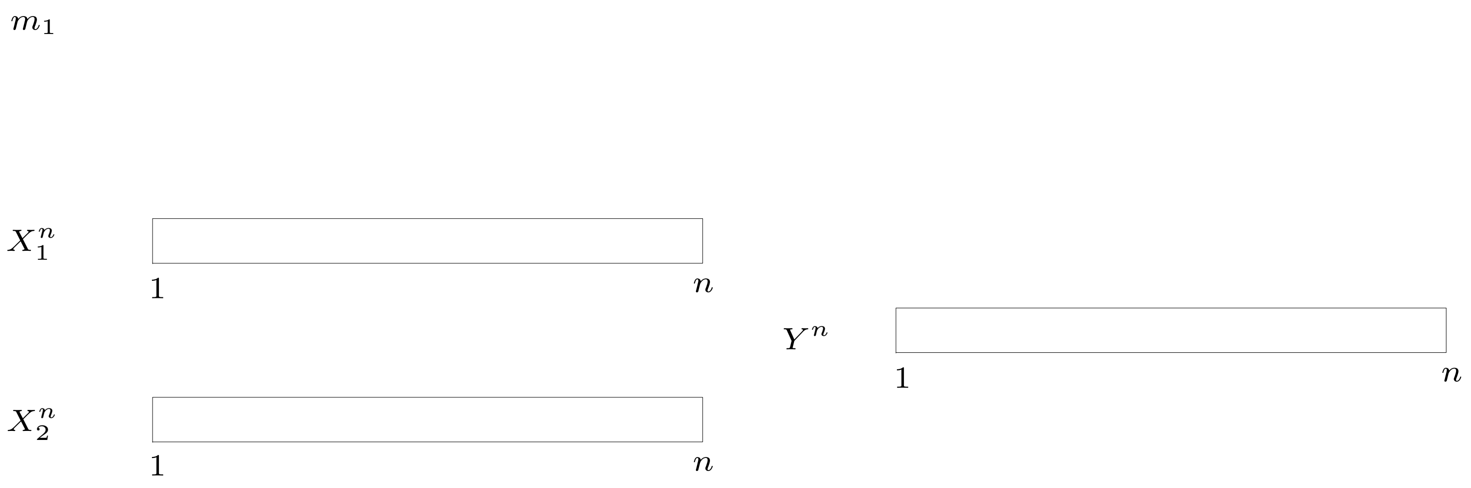
MAC with rate-limited feedback (cont'd)

Introduction

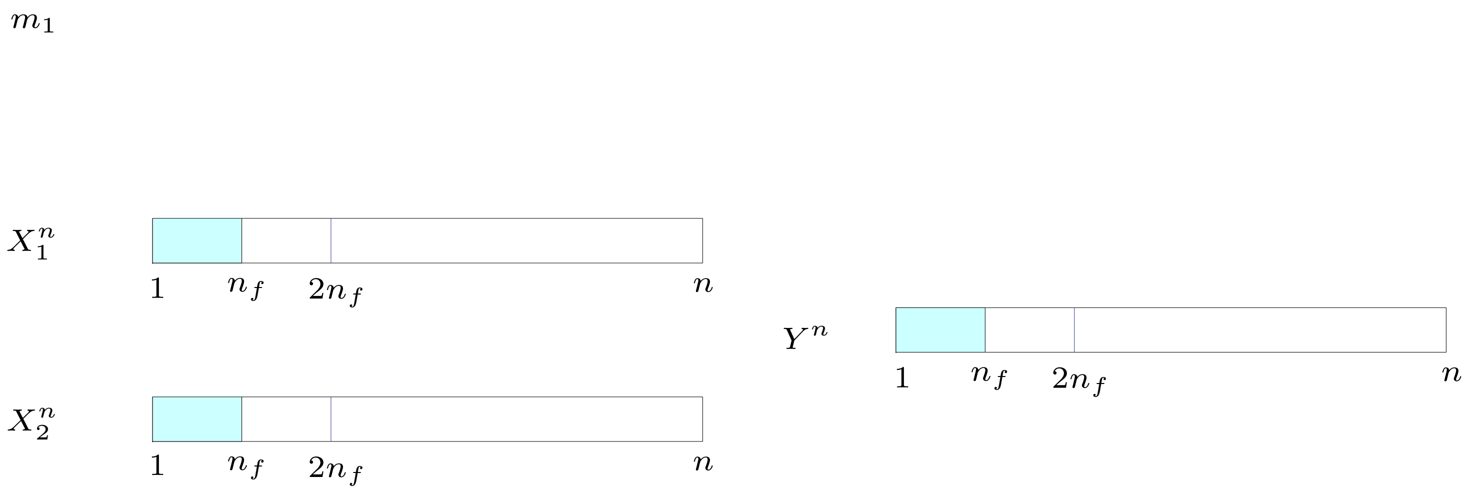
Channel and feedback models
- Perfect feedback
- Generalized feedback
- Rate-limited feedback

Main result

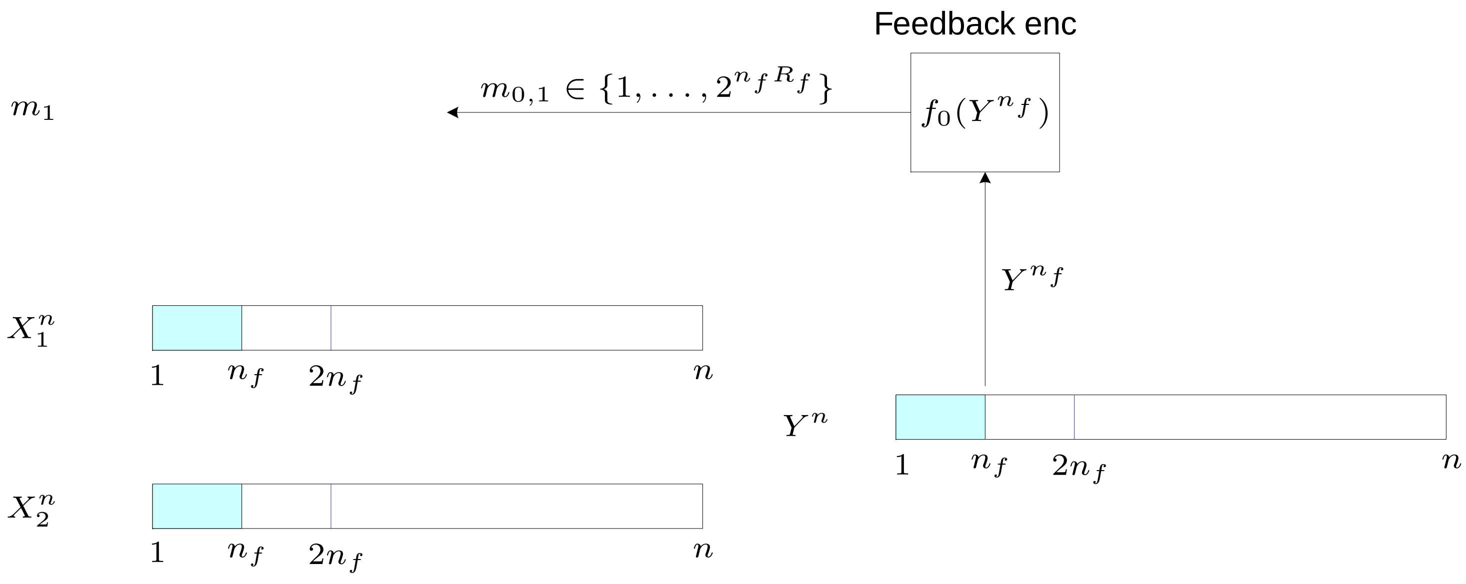
END



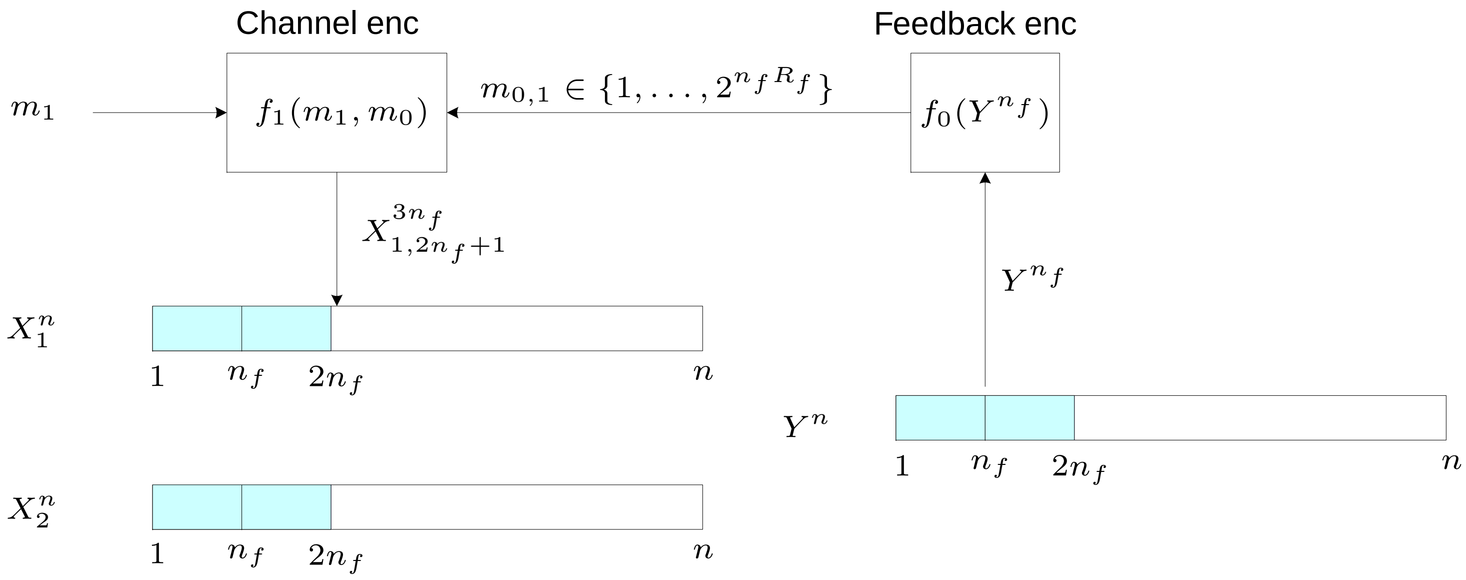
MAC with rate-limited feedback (cont'd)



MAC with rate-limited feedback (cont'd)

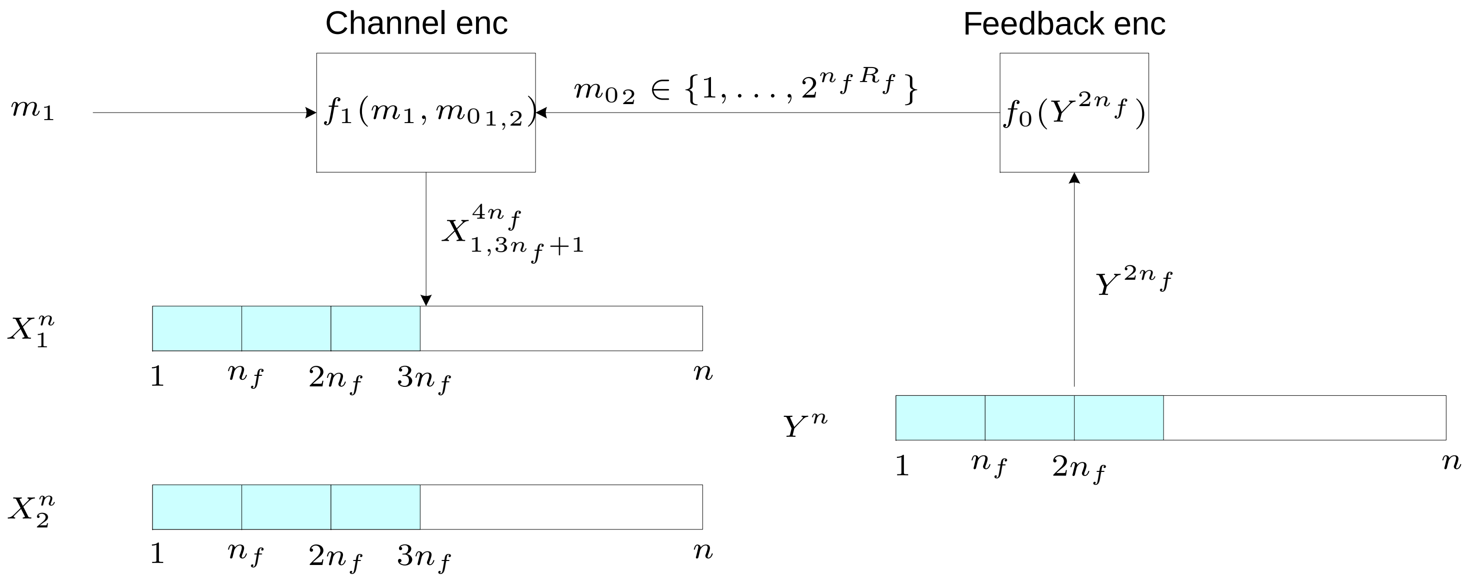


MAC with rate-limited feedback (cont'd)



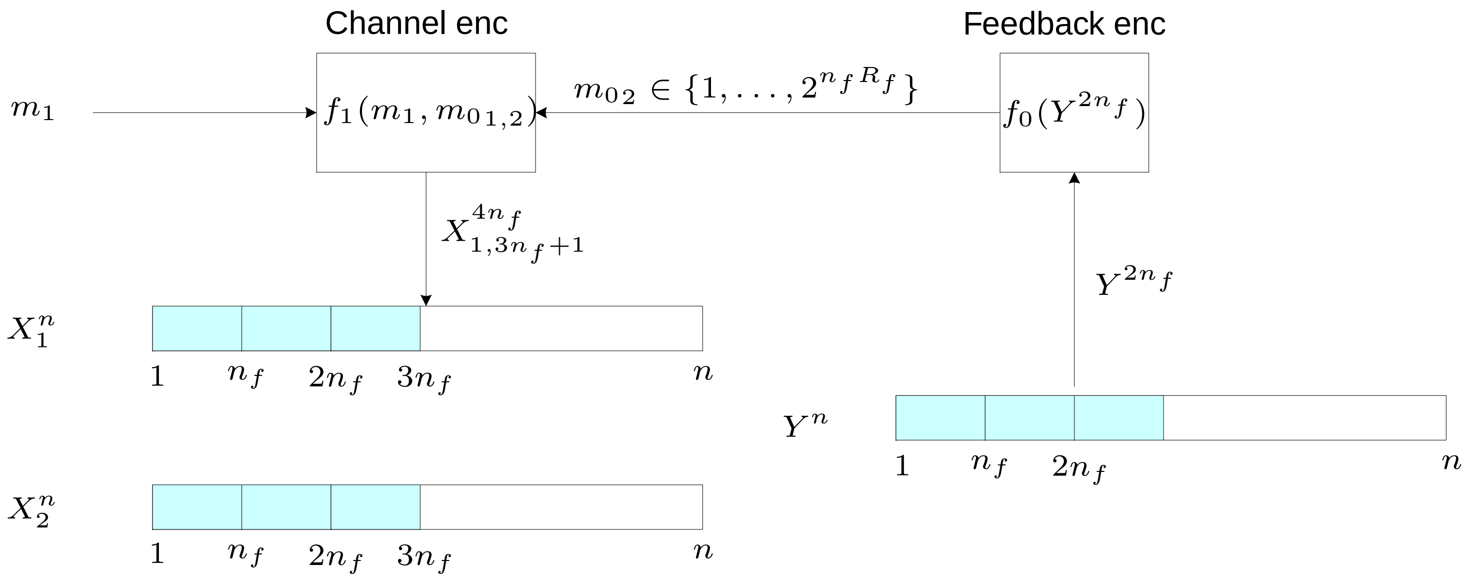
- An inherent two sub-blocks ($2n_f$) delay.

MAC with rate-limited feedback (cont'd)



- An inherent two sub-blocks ($2n_f$) delay.
- f_0 maps *all* the data it received thus far to one of $2^{n_f R_f}$ messages. Fixed rate feedback.

MAC with rate-limited feedback (cont'd)



- An inherent two sub-blocks ($2n_f$) delay.
- f_0 maps *all* the data it received thus far to one of $2^{n_f R_f}$ messages. Fixed rate feedback.
- The channel encoder maps m_1 and *all* the feedback messages $(m_{0,1}, m_{0,2}, \ldots)$ to the next input sub-block.

MAC with rate-limited feedback *(cont'd)*

Choose n, $n_f < n$. $L = \lfloor n/n_f \rfloor$. Define sets of messages

$$\mathcal{M}_k = \{1, \ldots, 2^{nR_k}\}, \;\; k = 1, 2, \quad \mathcal{M}_0 = \{1, \ldots, 2^{n_f R_f}\}.$$

MAC with rate-limited feedback (cont'd)

Choose n, $n_f < n$. $L = \lfloor n/n_f \rfloor$. Define sets of messages

$$\mathcal{M}_k = \{1, \ldots, 2^{nR_k}\}, \;\; k = 1, 2, \quad \mathcal{M}_0 = \{1, \ldots, 2^{n_f R_f}\}.$$

Definition: An $(R_1, R_2, n, n_f, R_f, \epsilon)$ code for a MAC with common fixed rate feedback of rate R_f, consists of encoding functions

$$f_{1,\ell} : \quad \mathcal{M}_1 \times \mathcal{M}_0^{[\ell-2]+} \to \mathcal{X}_1^{n_f},$$

$$f_{2,\ell} : \quad \mathcal{M}_2 \times \mathcal{M}_0^{[\ell-2]+} \to \mathcal{X}_2^{n_f}, \quad \ell = 1, 2, \ldots, L,$$

feedback encoding functions

$$f_{0,\ell} : \mathcal{Y}^{\ell n_f} \to \mathcal{M}_0, \;\; \ell = 1, 2, \ldots, L-2,$$

and a decoding function

$$g : \mathcal{Y}^n \to \mathcal{M}_1 \times \mathcal{M}_2,$$

with probability of error not exceeding ϵ

$$2^{-n(R_1+R_2)} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} P_e(m_1, m_2) \leq \epsilon,$$

MAC with rate-limited feedback (cont'd)

$P_e(m_1, m_2)$ – probability of error when the pair m_1, m_2 was sent

$$P_e(m_1, m_2) = \sum_{y^n:\ g(y^n) \neq (m_1, m_2)} P(y^n | \boldsymbol{f}_1(m_1, \boldsymbol{f}_0), \boldsymbol{f}_2(m_2, \boldsymbol{f}_0)).$$

$\boldsymbol{f}_0$, $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ – feedback encoder and two transmitter encoders:

$$\begin{aligned} \boldsymbol{f}_0 &= \boldsymbol{f}_0(y^n) = [m_{0,1}, m_{0,2}, \ldots, m_{0,L-1}] \\ &= \left[f_{0,1}(y^{n_f}), f_{0,2}(y^{2n_f}), \ldots, f_{0,L-1}(y^{(L-1)n_f})\right]. \end{aligned}$$

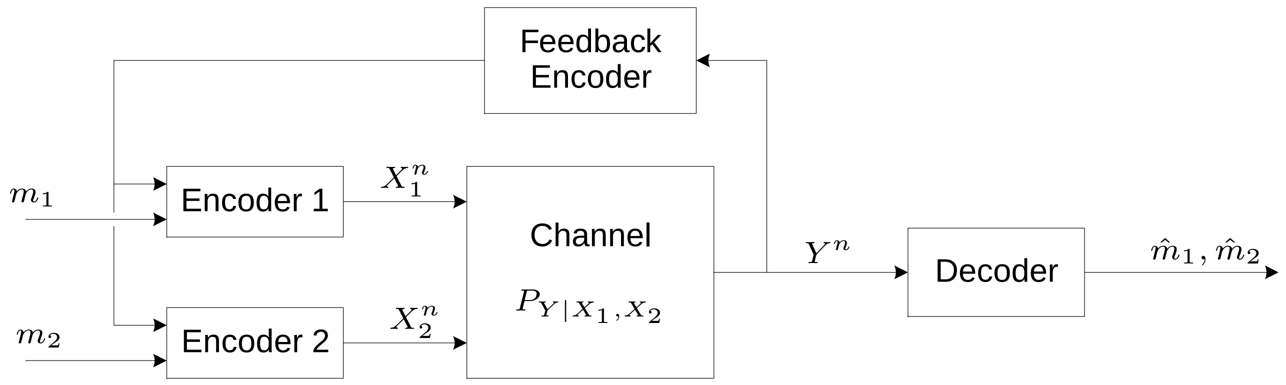
$$\begin{aligned} x_1^n = \boldsymbol{f}_1(m_1, \boldsymbol{f}_0(y^n)) = \\ & [f_{1,1}(m_1), f_{1,2}(m_1), f_{1,3}(m_1, f_{0,1}(y^{n_f})), \ldots, \\ & f_{1,\ell}(m_1, f_{0,\ell-2}(y^{(\ell-2)n_f})), \ldots]. \end{aligned}$$

MAC with rate-limited feedback *(cont'd)*

A triple (R_1, R_2, R_f) is *achievable* for the MAC with common fixed rate feedback, if for any $\epsilon > 0$ and sufficiently large n, there exist a feedback block length n_f and an $(R_1 - \epsilon, R_2 - \epsilon, n, n_f, R_f + \epsilon, \epsilon)$ code for this channel.

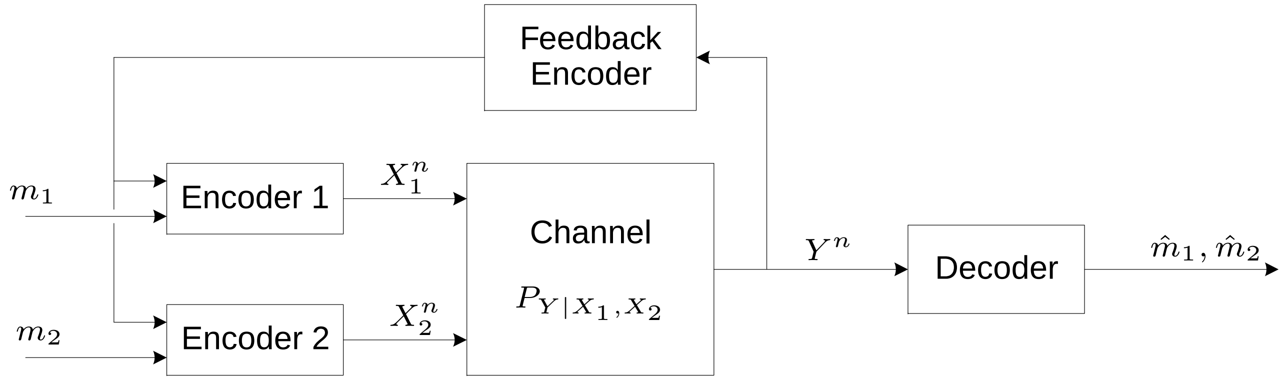
Main result

Coding scheme



Ingredients of coding scheme:

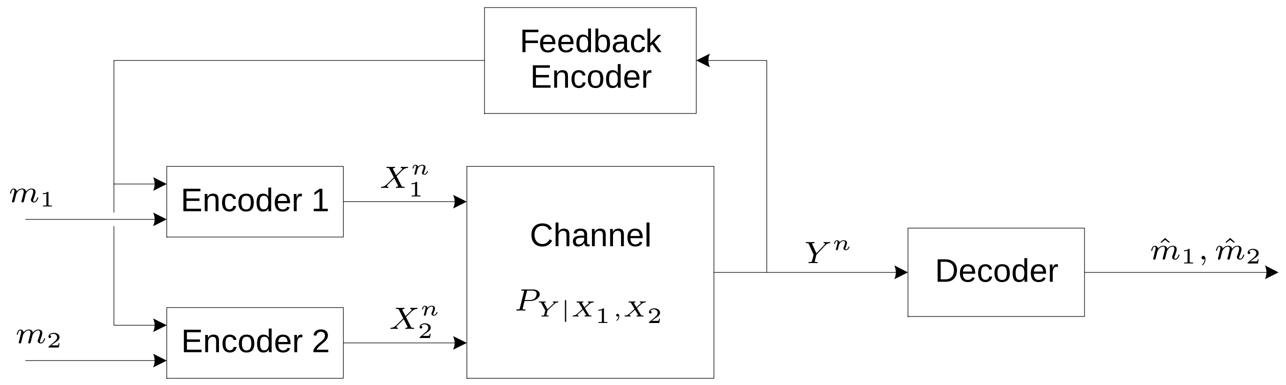
Coding scheme



Ingredients of coding scheme:

- Source coding with various levels of side information (Heegard and Berger, 1985)

Coding scheme

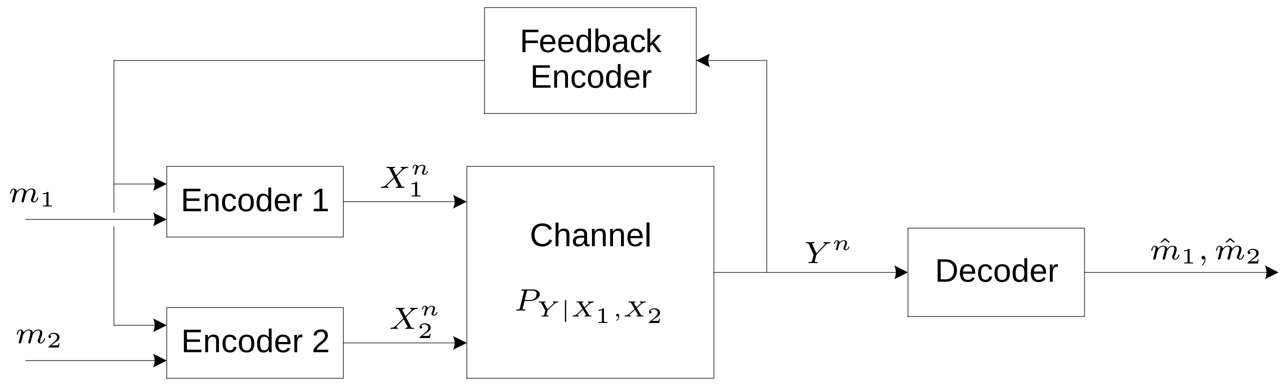


Ingredients of coding scheme:

- Source coding with various levels of side information (Heegard and Berger, 1985)
- Splitting the transmission rate: $R_1 = R_1' + (R_1 - R_1')$. R_1' can be decoded by user 2 via the feedback.

  Similarly for user 2 (Carleial 1981)

Coding scheme



Ingredients of coding scheme:

- Source coding with various levels of side information (Heegard and Berger, 1985)
- Splitting the transmission rate: $R_1 = R_1' + (R_1 - R_1')$. R_1' can be decoded by user 2 via the feedback.

  Similarly for user 2 (Carleial 1981)
- Super position of information and block Markov coding (Cover and Leung 1981, Carleial 1982)

Coding scheme (cont'd)

The following structure is a special case of the general definition:

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.
- At each block b, m_1 is represented by two indices:

$$j_1 \in 2^{nR_1'}, \quad k_2 \in 2^{n(R_1 - R_1')}.$$

Similarly for m_2 (indices j_2, k_2). The pairs (j_1, j_2) are partitioned into *bins*.

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.
- At each block b, m_1 is represented by two indices:

$$j_1 \in 2^{nR_1'}, \quad k_2 \in 2^{n(R_1 - R_1')}.$$

  Similarly for m_2 (indices j_2, k_2). The pairs (j_1, j_2) are partitioned into *bins*.
- R_f, R_1', R_2', and the feedback coding are designed so that user 1 knows $j_{2,b-2}$, and user 2 knows $j_{1,b-2}$ at end of block b.

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.
- At each block b, m_1 is represented by two indices:

$$j_1 \in 2^{nR_1'}, \quad k_2 \in 2^{n(R_1 - R_1')}.$$

  Similarly for m_2 (indices j_2, k_2). The pairs (j_1, j_2) are partitioned into *bins*.
- R_f, R_1', R_2', and the feedback coding are designed so that user 1 knows $j_{2,b-2}$, and user 2 knows $j_{1,b-2}$ at end of block b.
- Thus at the start of block b, each user knows i_b, the *bin number* of the pair $(j_{1,b-2}, j_{2,b-2})$.

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.
- At each block b, m_1 is represented by two indices:

$$j_1 \in 2^{nR_1'}, \quad k_2 \in 2^{n(R_1 - R_1')}.$$

Similarly for m_2 (indices j_2, k_2). The pairs (j_1, j_2) are partitioned into *bins*.

- R_f, R_1', R_2', and the feedback coding are designed so that user 1 knows $j_{2,b-2}$, and user 2 knows $j_{1,b-2}$ at end of block b.
- Thus at the start of block b, each user knows i_b, the *bin number* of the pair $(j_{1,b-2}, j_{2,b-2})$.
- i_b is sent *coherently* by the two users to the decoder, reducing its uncertainty about the message at block $b-2$.

Coding scheme *(cont'd)*

The following structure is a special case of the general definition:

- Transmission extends over $B+2$ blocks of size n. B pairs of messages (m_1, m_2) are decoded, $m_1 \in 2^{nR_1}$, $m_2 \in 2^{nR_2}$.
- At each block b, m_1 is represented by two indices:

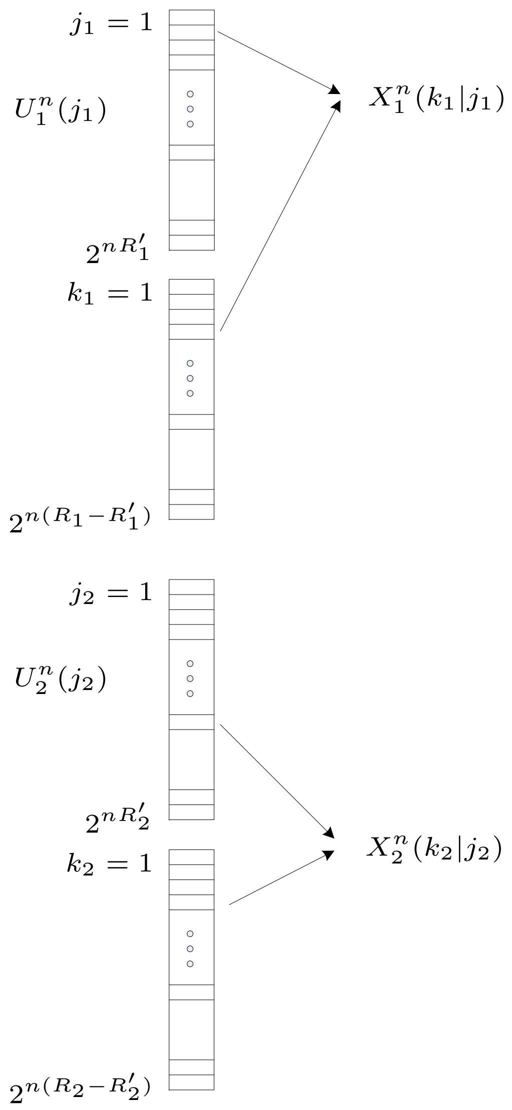
$$j_1 \in 2^{nR_1'}, \quad k_2 \in 2^{n(R_1 - R_1')}.$$

Similarly for m_2 (indices j_2, k_2). The pairs (j_1, j_2) are partitioned into *bins*.

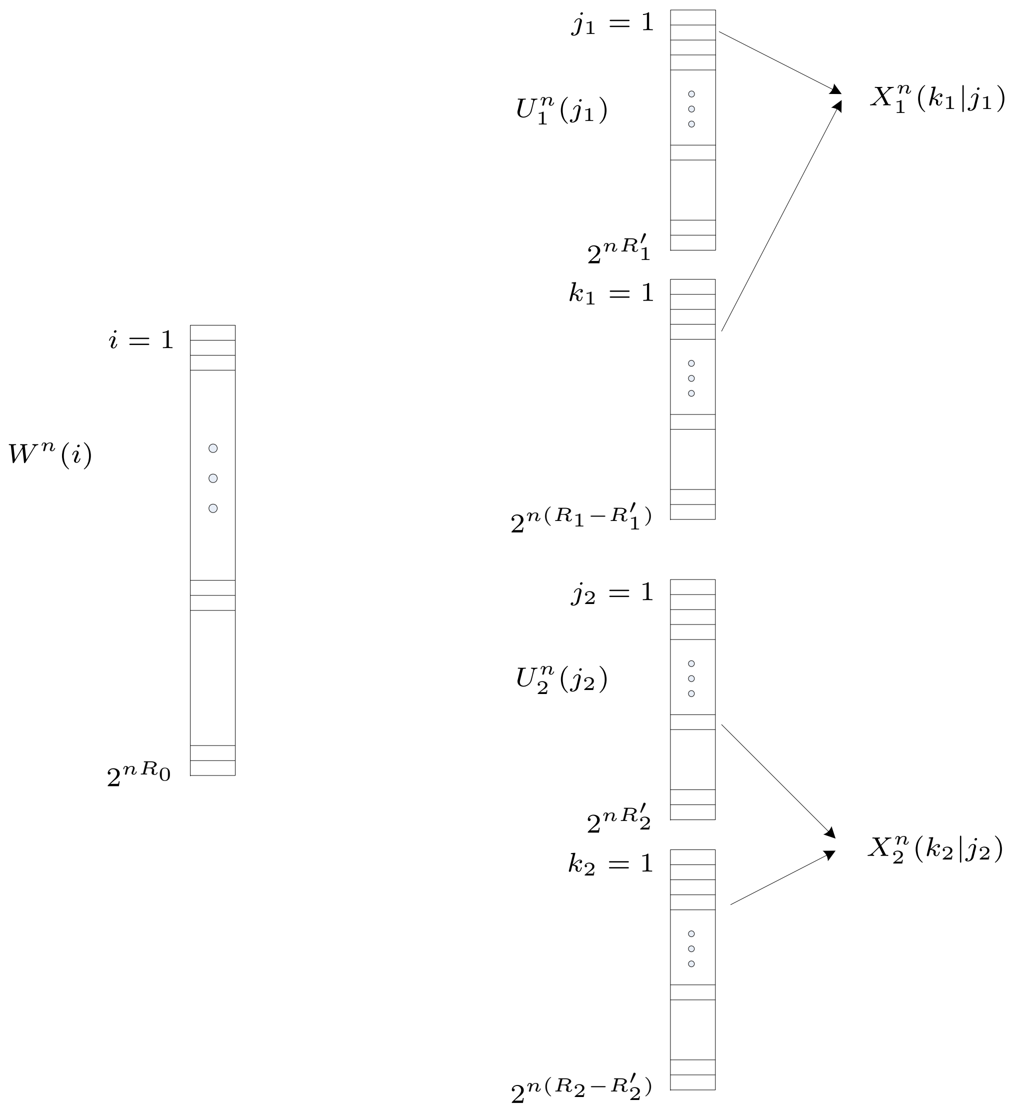
- R_f, R_1', R_2', and the feedback coding are designed so that user 1 knows $j_{2,b-2}$, and user 2 knows $j_{1,b-2}$ at end of block b.
- Thus at the start of block b, each user knows i_b, the *bin number* of the pair $(j_{1,b-2}, j_{2,b-2})$.
- i_b is sent *coherently* by the two users to the decoder, reducing its uncertainty about the message at block $b-2$.

The new ingredient relative to Carleial is the feedback coding

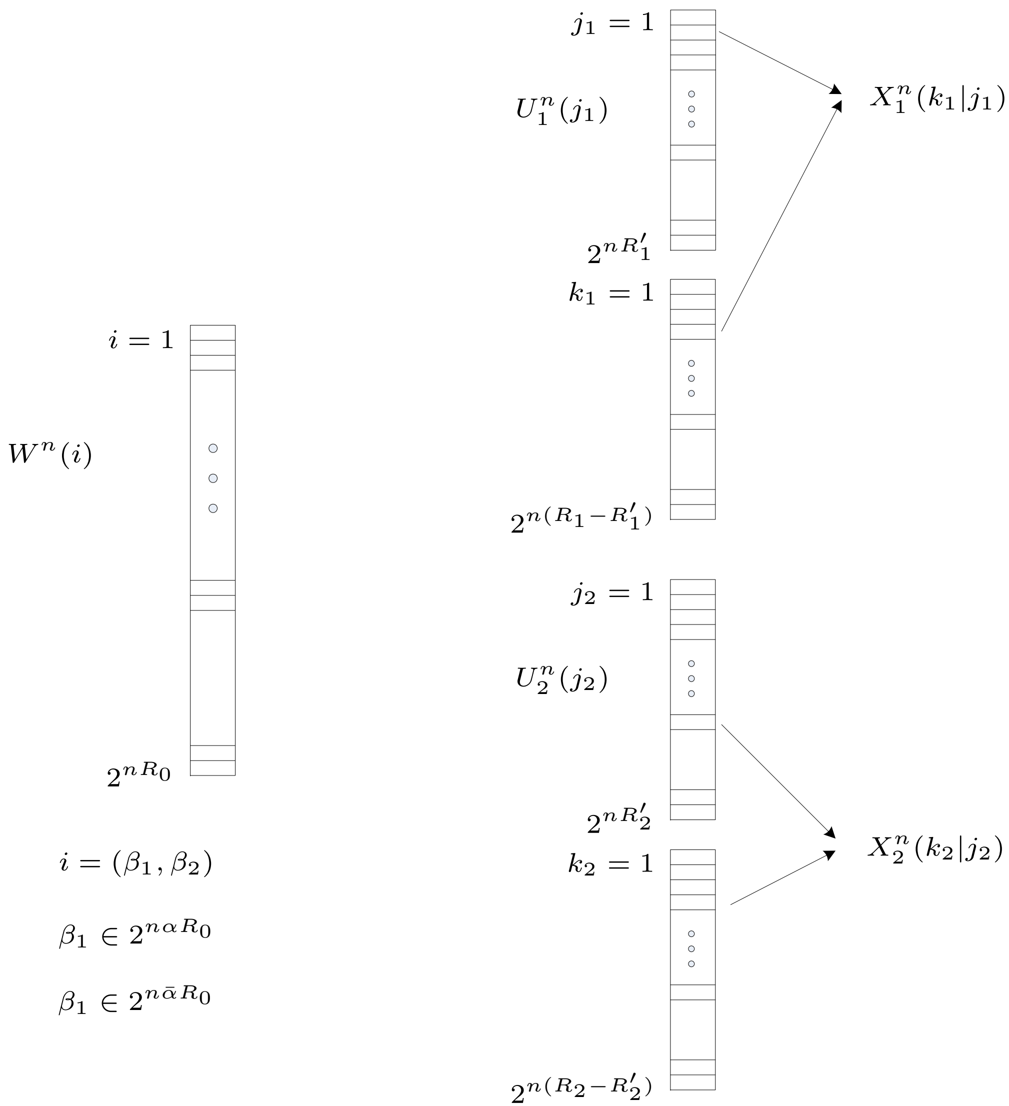
Coding scheme *(cont'd)*



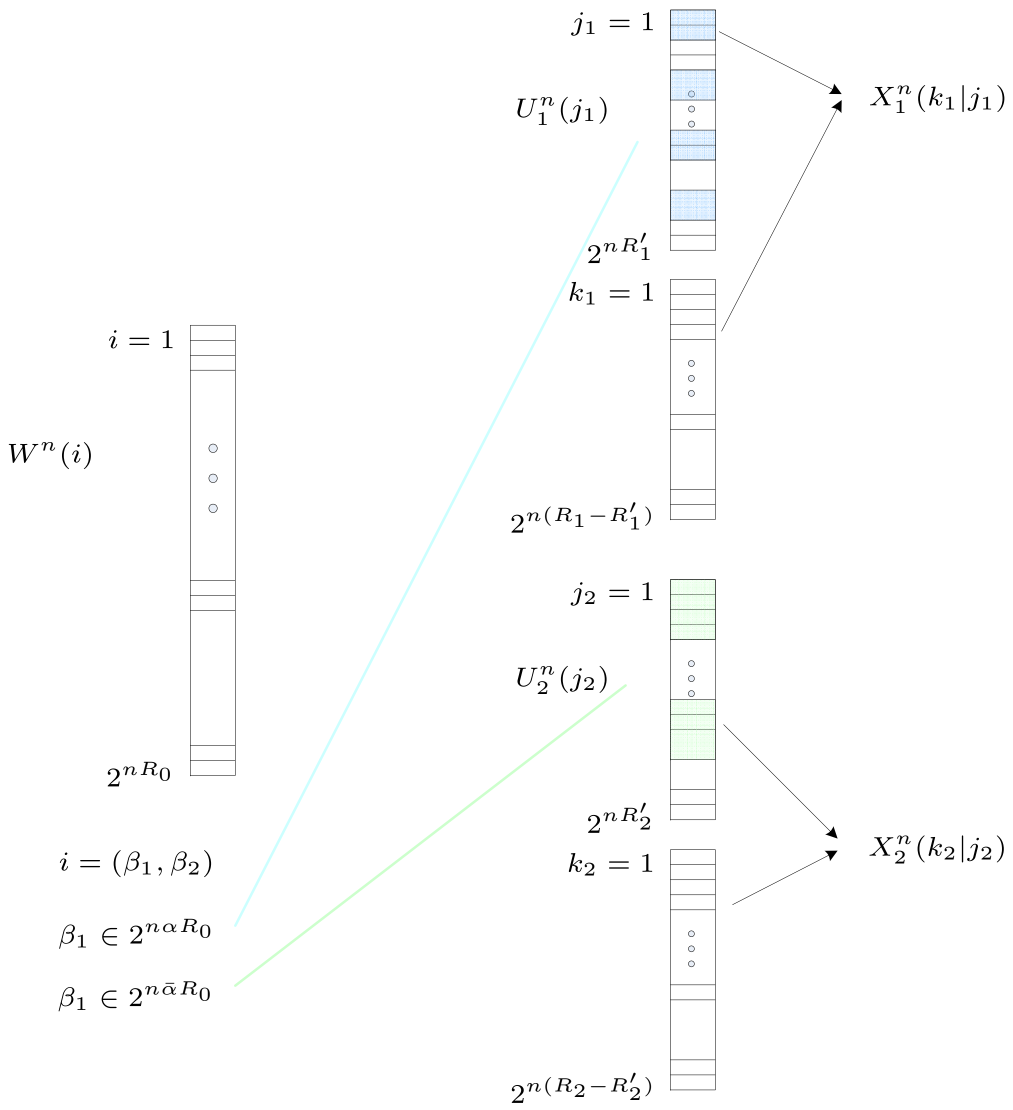
Coding scheme (cont'd)



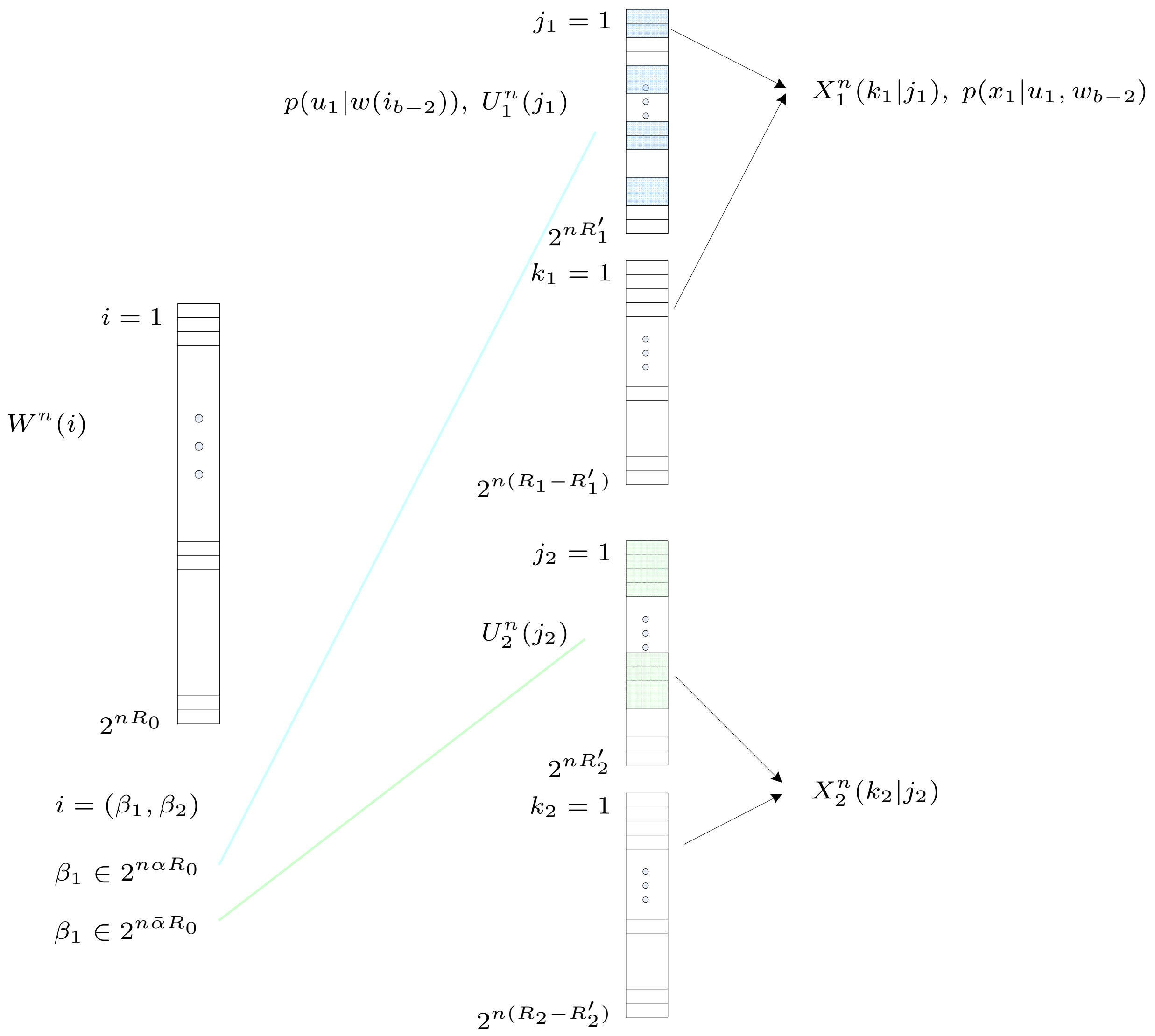
Coding scheme (cont'd)



Coding scheme *(cont'd)*



Coding scheme *(cont'd)*



Achievable region

Definition: Let $\mathcal{R}_i$ be the set of all (R_1, R_2, R_f) for which there exist $0 \le R_1' \le R_1$, $0 \le R_2' \le R_2$, and $\alpha \in [0,1]$, satisfying

$$\begin{aligned} R_f &\ge I(Y;Y_1|Y_{12},U_1,X_1,W) + I(Y;Y_2|Y_{12},U_2,X_2,W) \\ &\quad + \max\{I(Y;Y_{12}|U_1,X_1,W), I(Y;Y_{12}|U_2,X_2,W)\} \end{aligned}$$

$$\begin{aligned} R_1' &\le I(U_1;Y_2,Y_{12}|U_2,X_2,W) \\ R_2' &\le I(U_2;Y_1,Y_{12}|U_1,X_1,W) \end{aligned}$$

$$\begin{aligned} R_1' &\le I(U_1;Y|U_2,W) + \alpha I(W;Y) \\ R_2' &\le I(U_2;Y|U_1,W) + \bar{\alpha} I(W;Y) \\ R_1' + R_2' &\le I(W,U_1,U_2;Y) \end{aligned}$$

$$\begin{aligned} R_1 - R_1' &\le I(X_1;Y|W,U_1,U_2,X_2) \\ R_2 - R_2' &\le I(X_2;Y|W,U_1,U_2,X_1) \\ R_1 + R_2 - R_1' - R_2' &\le I(X_1,X_2;Y|W,U_1,U_2) \end{aligned}$$

for some $p(w)p(u_1|w)p(u_2|w)p(x_1|u_1,w)p(x_2|u_2,w)p(y|x_1,x_2)$
$p(y_{12}|y,w)p(y_1|y_{1,2},y,w)p(y_2|y_{1,2},y,w)$.

Achievable region *(cont'd)*

Theorem: For every memoryless finite alphabet MAC with feedback, $\mathcal{R}_i$ is achievable.

Achievable region *(cont'd)*

For perfect feedback, $\mathcal{R}_i$ reduces to the Cover-Leung region, with transmission rates R'_1 and R'_2.

$$
\begin{aligned}
R'_1 &\leq I(X_1; Y|W) \\
R'_2 &\leq I(X_2; Y|W) \\
R'_1 + R'_2 &\leq I(X_1, X_2; Y)
\end{aligned}
$$

for some $p(w)p(x_1|w)p(x_2|w)$

Obtained by substituting: $Y_{12} = Y_1 = Y_2 = Y$, $U_1 = X_1$, $U_2 = X_2$, and using the fact that Y is independent of W conditioned on X_1 and X_2.

Future work

- Compute $\mathcal{R}_i$ for specific models
- Extend to Bross-Lapidoth coding scheme.
- Variable rate feedback
- Outer bounds
- Separate feedback links

Thank You