The Broadcast Channel with Action Dependent States

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Problem formulation

The "regular" state-dependent BC:

Channel encoder:

\[ X_i = f(m_1, m_2, S^n) \quad \text{(non-causal SI)} \]
\[ X_i = f(m_1, m_2, S^i) \quad \text{(causal SI)} \]

\[ \frac{1}{n} \sum_{i=1}^{n} \Lambda(X_i) \leq \lambda \]

\[ P \left( (\hat{m}_1, \hat{m}_2) \neq (m_1, m_2) \right) \leq \epsilon \]
Problem formulation

Action-dependent states:

Encoding is performed in two parts:

- Given the pair of messages, an *action sequence* $A^n$ is created.
The actions generate a sequence of states $S^n$, via $P_{S|A}$. $S^n$ is available at the encoder (causally or noncausally).

- The encoder produces the channel input as a function of the messages and the states $S^n$. 
Problem formulation

Action-dependent states:

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\[ X_i = f(m_1, m_2, S^i) \quad \text{(causal SI)} \]
Motivation

- Controlling the channel: sometimes, the user can affect the channel statistics (state), albeit at a certain cost.
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- More specific channels: Channels (memories) with a rewrite option [Weissman 2010]:
  - The encoder makes a first pass and writes $A$ into the memory ($P_S|A = P_Y|X$).
  - After the first write, it observes the channel output ($S$), and decides accordingly about the form of the second write pass ($P_Y|\tilde{X}$).
  - $\tilde{X} = \{\text{noretire} \cup X\}$
  - Writing (first or second time) into a cell costs. Only 'norewrite' in free.
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  Writing (first or second time) into a cell costs. Only ‘norewrite’ in free.

- Cost of retrieving side information.
Motivation

Cost of retrieving SI:

In “regular” channel coding with SI, state is produced by nature (not by actions). It is either available at the encoder, or absent. No intermediate situation, and no cost on retrieving it.
Motivation

Cost of retrieving SI:
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Cost of retrieving SI:

- $S$ is produced by nature
- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.
**Motivation**

Cost of retrieving SI:

- $S$ is produced by nature
- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.
- The actions determine the availability (and quality) of side information at the encoder - $S_e$. 

$S \sim P_S$
Motivation

Cost of retrieving SI:

Probing capacity. Introduced in the context of single user channels by Asnani, Permuter, & Weissman, 2011.
Problem formulation

The basic setup:

- **Memoryless channel**
- **Causal SI:** $X_i = f(m_1, m_2, S^i)$
- **Cost on input and actions:**

$$\frac{1}{n} \sum_{i=1}^{n} \Lambda(A_i, X_i) \leq \lambda$$
Results on the same setup were obtained independently by Ahmedi and Simeone (ITW 2012, and http://arxiv.org/abs/1202.4438, Feb. 2012).
Previous results

Action dependent channels and sources

- Weissman 2010 - Introduced action dependent channels.
  - Capacity of single user channels, causal and non-causal models.
  - Bounds on the capacity of rewrite channels.
  - Connection to certain MAC models.

- H. Asnani, H. Permuter, & T. Weissman 2011 - Probing capacity: to observe or not to observe the side information? ($P_{S_e | S, A}$).

- Permuter & Weissman 2011 - Actions in the context of source coding: the side information vending machine

- Y.-K. Chia, H. Asnani, & T. Weissman 2011 (arXiv) - Multiterminal source coding with action dependent side information
Previous results

Action dependent single user channels

- Causal case (Weissman 2010):

\[
C = \max I(U, A; Y) \\
E[\Lambda(A, X)] \leq \lambda
\]

\[
P_{U,A|P_S|A}P_{X|S,U,A|P_Y|S,X}
\]
Previous results

Action dependent single user channels

- Causal case (Weissman 2010):

\[ C = \max I(U, A; Y) = I(A; Y) + I(U; Y|A) \]
\[ E[\Lambda(A, X)] \leq \lambda \]

\[ P_{U,A|P_S|A|P_X|S,U,A|P_Y|S,X} \]
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P_{U,A|P_S|A}P_{X|S,U,A}P_{Y|S,X}
\]

- Non causal case (Weissman 2010)\[
C = \max I(U, A; Y) - I(U; S|A)
\]
\[
E[\Lambda(A, X)] \leq \lambda
\]
\[
P_{A|P_S|A}P_{U|S,A}P_{X|S,U,A}P_{Y|S,X}
\]

In both cases, \(X\) can be taken to be a deterministic function of \((U, S)\).
Previous results

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  P_{U,A|S}P_{X|S,U,A}P_{Y|S,X}
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  P_{A|S}P_{U|S,A}P_{X|S,U,A}P_{Y|S,X}
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- Non causal case (Weisman 2010)

\[
C = \max I(U, A; Y) - I(U; S|A) = I(A; Y) + I(U; Y|A) - I(U; S|A) \\
E[\Lambda(A, X)] \leq \lambda
\]

\[
P_{A}P_{S|A}P_{U|S,A}P_{X|S,U,A}P_{Y|S,X}
\]

In both cases, \(X\) can be taken to be a deterministic function of \((U, S)\).
Previous results

State dependent broadcast channels

- S 2002, 2005 - Degraded, state dependent BC:
  - Capacity region for causal SI
  - Inner and outer bounds for non-causal SI
  - Capacity region for non-causal SI, where the stronger user is informed
Previous results

State dependent broadcast channels

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  - Capacity region for causal SI
  - Inner and outer bounds for non-causal SI
  - Capacity region for non-causal SI, where the stronger user is informed

- S & Shamai ISIT 2005:
  - Inner bounds for the general state dependent BC (Marton region + GP).
Previous results

State dependent BC

A state dependent BC \( P_{Y_1, Y_2|S,X} \) is called physically degraded if

\[
P_{Y_1, Y_2|S,X} = P_{Y_1|S,X} \cdot P_{Y_2|Y_1}
\]

and stochastically degraded if

\[
P_{Y_2|S,X}(y_2|s,x) = \sum_{y_1} P_{Y_1, Y_2|S,X}(y_1, y_2|s,x) \cdot W_{Y_2|Y_1}(y_2|y_2)
\]

for some \( W_{Y_2|Y_1} \).
Main results

- Memoryless channel
- Causal SI: \( X_i = f(m_1, m_2, S^i) \)
- Cost on input and actions:

\[
\frac{1}{n} \sum_{i=1}^{n} \Lambda(A_i, X_i) \leq \lambda \quad (\Lambda, \lambda \in \mathbb{R}^d)
\]
Main results

- Capacity region: $C_c$
- $C_c$ depends on $P_{Y_1,Y_2|S,X}$ only via $P_{Y_1|S,X}$ and $P_{Y_2|S,X}$. 
  ⇒ No distinction has to be made between physically and stochastically degraded channels. General term: *degraded*. 
Main results

The causal case

\( \mathcal{R}_c \) - the collection of all \((\lambda, R_1, R_2)\) such that

\[
R_1 \leq I(U, A; Y_1|K) \\
R_2 \leq I(K; Y_2) \\
E[\Lambda_k(A, X)] \leq \lambda_k, \quad k = 1, 2, \ldots, d
\]

for some

\[
P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}.
\]

Theorem

For the degraded BC with action dependent states and causal SI

\[
\mathcal{C}_c = \mathcal{R}_c.
\]
Main results

Properties of $\mathcal{R}_c$

\[ R_1 \leq I(U, A; Y_1 | K) \]
\[ R_2 \leq I(K; Y_2) \]
\[ E [\Lambda_k(A, X)] \leq \lambda_k, \quad k = 1, 2, \ldots, d \]

\[ P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}. \]

- $\mathcal{R}_c$ is convex.
Main results
Properties of $\mathcal{R}_c$

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\[ P_{A,K,U,S,X,Y,Z} = P_{K, U} P_{A|K, U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X} . \]

- $\mathcal{R}_c$ is convex.
- To exhaust $\mathcal{R}_c$, $P_{A|K,U}$ and $P_{X|A,K,U,S}$ can be 0−1 laws.
Main results

Properties of $\mathcal{R}_c$

\[ R_1 \leq I(U, A; Y_1|K) \]
\[ R_2 \leq I(K; Y_2) \]
\[ \mathbb{E} \left[ \Lambda_k(A, X) \right] \leq \lambda_k, \quad k = 1, 2, \ldots, d \]

\[ P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X} \]

- $\mathcal{R}_c$ is convex.
- To exhaust $\mathcal{R}_c$, $P_{A|K,U}$ and $P_{X|A,K,U,S}$ can be 0−1 laws. Can drop the $A$ from the bound on $R_1$. 

▶ Bounds on alphabets
\[ |K| \leq |S||A||X| + 1 \]
\[ |U| \leq |S||A||X| \left( |S||A||X| + 1 \right) \]
Main results

Properties of $\mathcal{R}_c$

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R_1 \leq I(U, A; Y_1 | K)
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P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X}.
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- Bounds on alphabets

\[
|K| \leq |S||A||X| + 1
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|U| \leq |S||A||X| (|S||A||X| + 1)
\]
Main results

Extended Shannon strategies

- Shannon strategies $\mathcal{T} = \{ t | t : S \to \mathcal{X} \}$. $T$ - random strategy.
Main results

Extended Shannon strategies

- Shannon strategies $\mathcal{T} = \{ t | t : S \rightarrow \mathcal{X} \}$. $T$ - random strategy.
- Define a derived channel $P_{Y_1|A,T}$ as follows. First

$$P_{Y_1|A,S,T}(y_1|a,s,t) = P_{Y_1|A,S,X}(y_1|a,s,t(s)) = P_{Y_1|S,X}(y_1|s,t(s))$$
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\]

then

\[
P_{Y_1|A,T}(y_1|a,t) = \sum_s P_{Y_1|A,S,T}(y_1|a,s,t)P_{S|A}(s|a) = \sum_s P_{Y_1|S,X}(y_1|s,t(s))P_{S|A}(s|a)
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- $I(U, A; Y_1|K)$ can be replaced by $I(A, T; Y_1|K)$
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$$P_{Y_1|A,T}(y_1|a,t) = \sum_s P_{Y_1|A,S,T}(y_1|a,s,t)P_S|A(s|a)$$

$$= \sum_s P_{Y_1|S,X}(y_1|s,t(s))P_S|A(s|a)$$

- $I(U,A; Y_1|K)$ can be replaced by $I(A, T; Y_1|K)$
- Extended strategies $\mathcal{V} = \mathcal{A} \times \mathcal{T}$. $\mathcal{V}$ - random extended strategy.
Main results

Extended Shannon strategies

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  $$= P_{Y_1|S,X}(y_1|s,t(s))$$
  
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- $I(U,A;Y_1|K)$ can be replaced by $I(A,T;Y_1|K)$
- Extended strategies $\mathcal{V} = \mathcal{A} \times \mathcal{T}$. $\mathcal{V}$ - random extended strategy.
- The bound on $R_1$ becomes $R_1 \leq I(\mathcal{V}; Y|K)$. 
Main results

Proof technique

▶ Single user channel:

- An action sequence $A_n(m)$ is generated for every message $m$. The actions generate the state sequence $S_n$.
- A codeword $U_n(m)$ is generated independently of $A_n$. Joint typicality of $U_n$ and $(A_n, S_n)$ is guaranteed due to $U - A - S$.
- Non-causal case: A codebook $U_n(j, m)$ is generated for every $M$. Encoder looks for an index $j$ such that $(U_n(j, m), S_n, A_n(m))$ are jointly typical.

▶ BC: In the problem formulation, the action depends on both messages, $m_1$ and $m_2$. Cannot start with $A_n(m_1, m_2)$. Running into problems with the satellite coding.
Main results

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  - An action sequence $A^n(m)$ is generated for every message $m$. The actions generate the state sequence $S^n$. 

Previous results

Main results

Causal SI
Properties of $\mathcal{R}_C$
Proof technique
Probing capacity
Action feedback
Non-causal SI

Summary & future work
Main results

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- Cannot start with $A^n(m_1, m_2)$. Running into problems with the satellite coding.
Main results

Proof technique

\[ R_1 \leq I(U, A; Y_1|K) \]
\[ R_2 \leq I(K; Y_2) \]

\[ P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}. \]
Main results

Proof technique

\[ R_1 \leq I(U, A; Y_1|K) \]
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P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}.
\]

- Generate a codeword \( K^n(m_2) \).
Main results

Proof technique

\[ R_1 \leq I(U, A; Y_1 | K) \]
\[ R_2 \leq I(K; Y_2) \]

\[ P_{A,K,U,S,X,Y,Z} = P_K,UP_{A|K},UP_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}. \]

- Generate a codeword \( K^n(m_2) \).
- Generate actions \( A^n(m_1, m_2) \) by \( \prod_{i=1}^{n} P_{A|K}(\cdot | K_i(m_2)) \).
Main results

Proof technique

\[ R_1 \leq I(U, A; Y_1|K) \]
\[ R_2 \leq I(K; Y_2) \]

\[ P_{A, K, U, S, X, Y, Z} = P_K, U P_{A|K, U} P_X|A, K, U, S P_S|A P_{Y_1, Y_2|S, X}. \]

- Generate a codeword \( K^n(m_2) \).
- Generate actions \( A^n(m_1, m_2) \) by \( \prod_{i=1}^n P_{A|K} (\cdot | K_i(m_2)) \)
- Generate a codeword \( U^n(m_1|m_2) \) by \( \prod_{i=1}^n P_{U|K, A} (\cdot | K_i(m_2), A_i(m_1, m_2)) \)
Main results

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- Generate a codeword \( U^n(m_1|m_2) \) by \( \prod_{i=1}^n P_{U|K,A}(\cdot|K_i(m_2), A_i(m_1, m_2)) \)

Can switch the order of \( A^n \) and \( U^n \).
Main results

Probing capacity

\[ S \sim P_S \]

\[ A^n \rightarrow P_{S_e|S,A} \rightarrow S^n \rightarrow X^n \rightarrow \text{Channel} \]

\[ \text{Encoder} \]

\[ S_e^n \rightarrow \text{Decoder 1} \rightarrow \hat{m}_1 \]

\[ m_1, m_2 \rightarrow \text{Encoder} \]

\[ m_1, m_2 \rightarrow \text{Encoder} \]

\[ Y_1^n, Y_2^n \rightarrow \text{Decoders} \]

\[ \hat{m}_2 \]
Main results

Probing capacity

\[ C_c = \bigcup \{ (\lambda, R_1, R_2) : R_1 \leq I(U, A; Y_1 | K) \]
\[ R_2 \leq I(K; Y_2) \]
\[ E [\Lambda_k(A, X)] \leq \lambda_k, \quad k = 1, 2, \ldots, d \} \]

Union over

\[ P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S_e}P_{S_e|A,S}P_{Y_1,Y_2|S,X}. \]
Main results

Action feedback

Q: What happens if the actions depend on past values of the states?

\[ A_i = A_i(m_1, m_2, S^{i-1}) \]
Main results

Action feedback

Q: What happens if the actions depend on past values of the states?

In single user channels, capacity is not increased (both cases, $X_i(m, S^i)$ or $X_i(m, S^n)$).
Main results

Action feedback

Theorem

*The capacity region of the physically degraded broadcast channel with action-dependent states known causally at the encoder is not increased if the actions depend on past values of the states.*
Main results
Action feedback

Theorem

The capacity region of the physically degraded broadcast channel with action-dependent states known causally at the encoder is not increased if the actions depend on past values of the states.

Action feedback does increase the capacity region of the stochastically degraded broadcast channel with action dependent states.
Main results

Action feedback

Theorem

The capacity region of the physically degraded broadcast channel with action-dependent states known causally at the encoder is not increased if the actions depend on past values of the states.

Action feedback does increase the capacity region of the stochastically degraded broadcast channel with action dependent states.

Resembles the situation in BC without states (El Gamal 78, Dueck 80).
Main results

Non-causal SI, informed strong decoder

- Even without actions, the state-dependent degraded BC with non-causal SI is still an open problem.
- Solved for the case where the stronger user is informed.
Main results

Non-causal SI, informed strong decoder

An achievable region:

\[ R_1 \leq I(X; Y_1|K, A) \]
\[ R_2 \leq I(K, A; Y_2) - I(K; S|A) \]
\[ E[\Lambda(A, X)] \leq \lambda \]

\[ P_A P_{S|A} P_{K|S,A} P_{X|S,K,A} P_{Y_1,Y_2|S,X} \]
Summary

- The degraded BC with action-dependent states and causal SI is solved.
- Future work: Non-causal setting. Good examples.