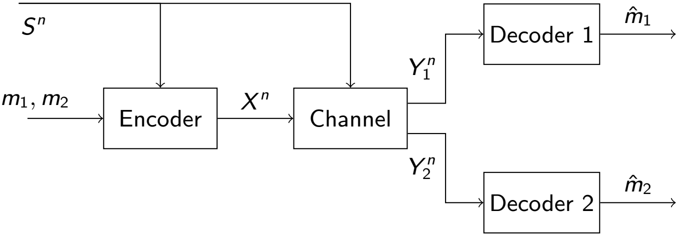
The Broadcast Channel with Action Dependent States

Yossef Steinberg and Tsachy Weissman

ISIT 2012

Problem formulation

The "regular" state-dependent BC:



- Channel encoder:

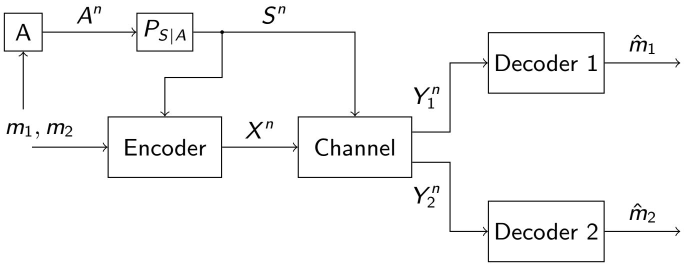
$$X_i = f(m_1, m_2, S^n) \quad \text{(non-causal SI)}$$
$$X_i = f(m_1, m_2, S^i) \quad \text{(causal SI)}$$

- $\frac{1}{n}\sum_{i=1}^{n} \Lambda(X_i) \leq \lambda$

$$P\left((\hat{m}_1, \hat{m}_2) \neq (m_1, m_2)\right) \leq \epsilon$$

Problem formulation

Action-dependent states:



Encoding is performed in two parts:

- Given the pair of messages, an *action sequence* A^n is created.
  The actions generate a sequence of states S^n, via $P_{S|A}$.
  S^n is available at the encoder (causally or noncausally).
- The encoder produces the channel input as a function of the messages and the states S^n.

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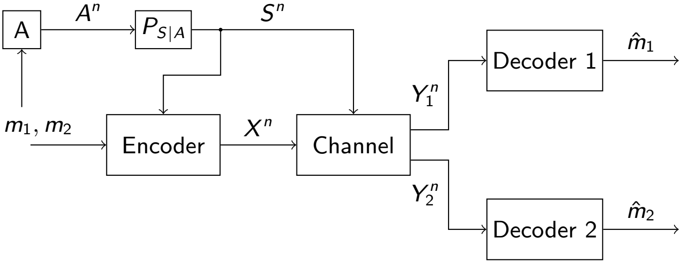

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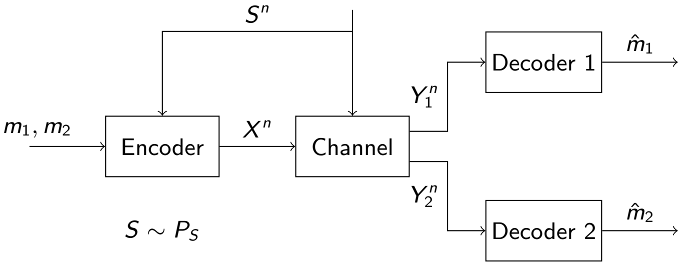
- Cost of retrieving side information.

Motivation

Cost of retrieving SI:



In "regular" channel coding with SI, state is produced by nature (not by actions). It is either *available* at the encoder, or *absent*. No intermediate situation, and no cost on retrieving it.

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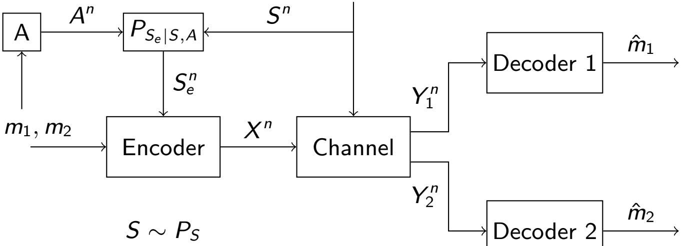

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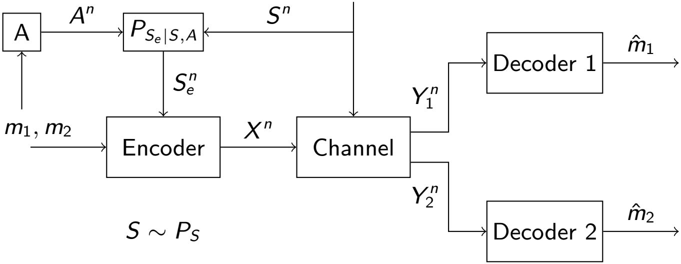

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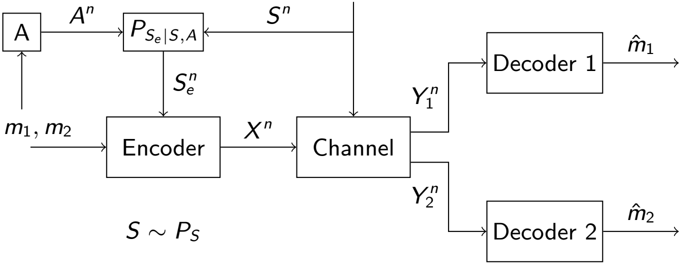

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- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.

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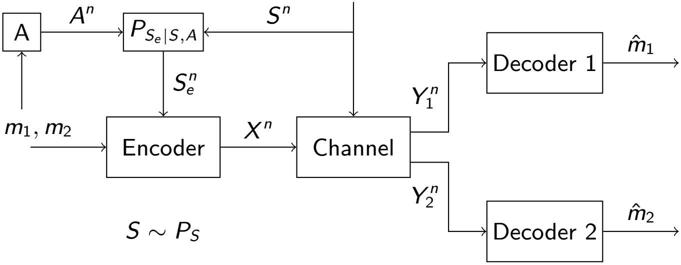

- S is produced by nature
- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.
- The actions determine the availability (and quality) of side information at the encoder - S_e.

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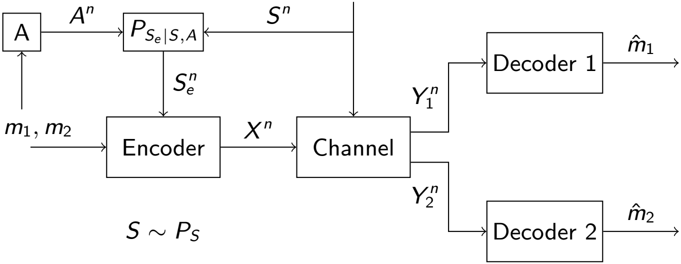
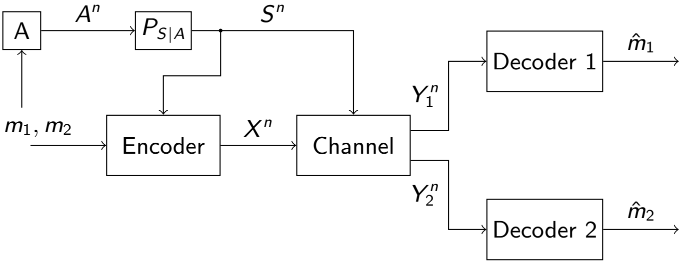


Probing capacity. Introduced in the context of single user channels by Asnani, Permuter, & Weissman, 2011.

Problem formulation

The basic setup:



- Memoryless channel
- Causal SI: $X_i = f(m_1, m_2, S^i)$
- Cost on input and actions:

$$\frac{1}{n}\sum_{i=1}^{n} \Lambda(A_i, X_i) \leq \lambda$$

Results on the same setup were obtained independently by Ahmedi and Simeone
(ITW 2012, and `http://arxiv.org/abs/1202.4438`, Feb. 2012).

Previous results

Action dependent channels and sources

- Weissman 2010 - Introduced action dependent channels.
  - Capacity of single user channels, causal and non-causal models.
  - Bounds on the capacity of rewrite channels.
  - Connection to certain MAC models.
- H. Asnani, H. Permuter, & T. Weissman 2011 - Probing capacity: to observe or not to observe the side information? ($P_{S_e|S,A}$).
- Permuter & Weissman 2011 - Actions in the context of source coding: the side information vending machine
- Y.-K. Chia, H. Asnani, & T. Weissman 2011 (arXiv) - Multiterminal source coding with action dependent side information

Previous results

Action dependent single user channels

- Causal case (Weissman 2010):

$$C = \max I(U, A; Y)$$
$$\mathrm{E}\left[\Lambda(A, X)\right] \leq \lambda$$

$$P_{U,A} P_{S|A} P_{X|S,U,A} P_{Y|S,X}$$

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$$C = \max_{\substack{P_{U,A}P_{S|A}P_{X|S,U,A}P_{Y|S,X} \\ \mathrm{E}[\Lambda(A,X)] \le \lambda}} I(U,A;Y) = I(A;Y) + I(U;Y|A)$$

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- Non causal case (Weisman 2010)

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In both cases, X can be taken to be a deterministic function of (U, S).

Previous results

State dependent broadcast channels

- S 2002, 2005 - Degraded, state dependent BC:
  - Capacity region for causal SI
  - Inner and outer bounds for non-causal SI
  - Capacity region for non-causal SI, where the stronger user is informed

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- S & Shamai ISIT 2005:
  - Inner bounds for the general state dependent BC (Marton region + GP).

Previous results

State dependent BC

A state dependent BC $P_{Y_1,Y_2|S,X}$ is called physically degraded if

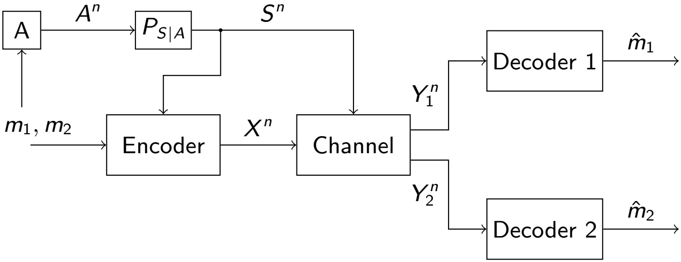
$$P_{Y_1,Y_2|S,X} = P_{Y_1|S,X} \cdot P_{Y_2|Y_1}$$

and stochastically degraded if

$$P_{Y_2|S,X}(y_2|s,x) = \sum_{y_1} P_{Y_1,Y_2|S,X}(y_1,y_2|s,x) \cdot W_{Y_2|Y_1}(y_2|y_2)$$

for *some* $W_{Y_2|Y_1}$.

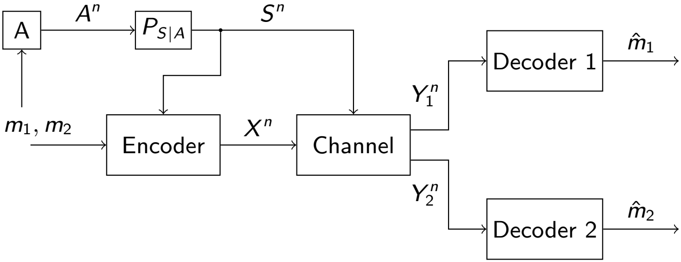
Main results



- Memoryless channel
- Causal SI: $X_i = f(m_1, m_2, S^i)$
- Cost on input and actions:

$$\frac{1}{n}\sum_{i=1}^{n} \Lambda(A_i, X_i) \leq \lambda \qquad (\Lambda,\ \lambda \in R^d)$$

Main results



- Capacity region: $\mathcal{C}_c$
- $\mathcal{C}_c$ depends on $P_{Y_1,Y_2|S,X}$ only via $P_{Y_1|S,X}$ and $P_{Y_2|S,X}$.

  $\Rightarrow$ No distinction has to be made between physically and stochastically degraded channels. General term: *degraded*.

Main results

The causal case

$\mathcal{R}_c$ - the collection of all (λ, R_1, R_2) such that

$$R_1 \leq I(U, A; Y_1|K)$$
$$R_2 \leq I(K; Y_2)$$
$$\mathrm{E}\left[\Lambda_k(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d$$

for some

$$P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X}.$$

Theorem

For the degraded BC with action dependent states and causal SI

$$\mathcal{C}_c = \mathcal{R}_c.$$

Main results

Properties of $\mathcal{R}_c$

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- Bounds on alphabets

$$|\mathcal{K}| \le |\mathcal{S}||\mathcal{A}||\mathcal{X}| + 1$$
$$|\mathcal{U}| \le |\mathcal{S}||\mathcal{A}||\mathcal{X}| \left(|\mathcal{S}||\mathcal{A}||\mathcal{X}| + 1\right)$$

Main results

Extended Shannon strategies

- Shannon strategies $\mathcal{T} = \{t | t : \mathcal{S} \rightarrow \mathcal{X}\}$. T - random strategy.

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$$\begin{aligned} P_{Y_1|A,S,T}(y_1|a,s,t) &= P_{Y_1|A,S,X}(y_1|a,s,t(s)) \\ &= P_{Y_1|S,X}(y_1|s,t(s)) \end{aligned}$$

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- $I(U, A; Y_1|K)$ can be replaced by $I(A, T; Y_1|K)$
- Extended strategies $\mathcal{V} = \mathcal{A} \times \mathcal{T}$. V - random extended strategy.
- The bound on R_1 becomes $R_1 \le I(V; Y|K)$.

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Proof technique

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- BC: In the problem formulation, the action depends on both messages, m_1 and m_2.
  - Cannot start with $A^n(m_1, m_2)$. Running into problems with the satellite coding.

Main results

Proof technique

$$R_1 \leq I(U, A; Y_1|K)$$
$$R_2 \leq I(K; Y_2)$$

$$P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X}.$$

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- Generate a codeword $U^n(m_1|m_2)$ by $\prod_{i=1}^n P_{U|K,A}(\cdot|K_i(m_2), A_i(m_1, m_2))$

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$$R_1 \le I(U, A; Y_1|K)$$
$$R_2 \le I(K; Y_2)$$

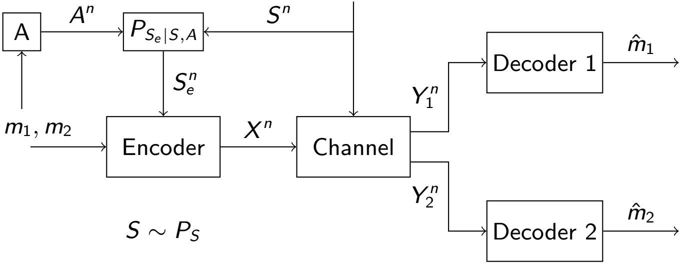
$$P_{A,K,U,S,X,Y,Z} = P_{K,U}P_{A|K,U}P_{X|A,K,U,S}P_{S|A}P_{Y_1,Y_2|S,X}.$$

- Generate a codeword $K^n(m_2)$.
- Generate actions $A^n(m_1, m_2)$ by $\prod_{i=1}^n P_{A|K}(\cdot|K_i(m_2))$
- Generate a codeword $U^n(m_1|m_2)$ by $\prod_{i=1}^n P_{U|K,A}(\cdot|K_i(m_2), A_i(m_1, m_2))$

Can switch the order of A^n and U^n.

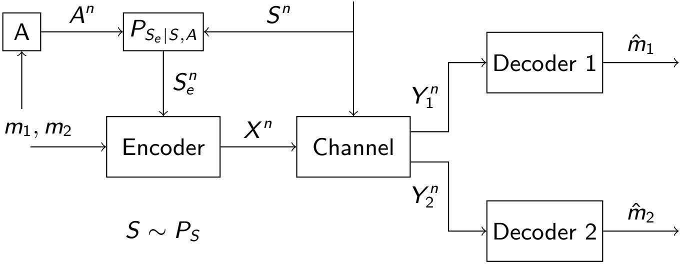
Main results

Probing capacity



Main results

Probing capacity



$$\mathcal{C}_c = \bigcup \{(\lambda, R_1, R_2) : \quad R_1 \leq I(U, A; Y_1|K) \quad R_2 \leq I(K; Y_2) \quad \mathrm{E}\left[\Lambda_k(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d\}$$

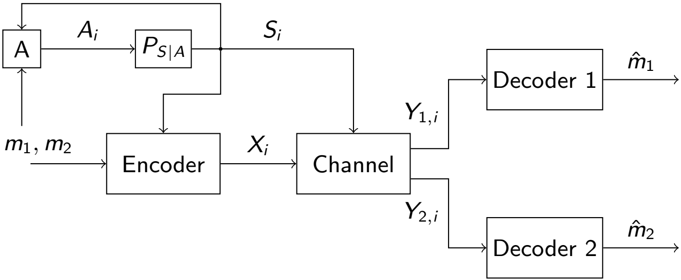
Union over

$$P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S_e} P_{S_e|A,S} P_{Y_1,Y_2|S,X}.$$

Main results

Action feedback

Q: What happens if the actions depend on past values of the states?

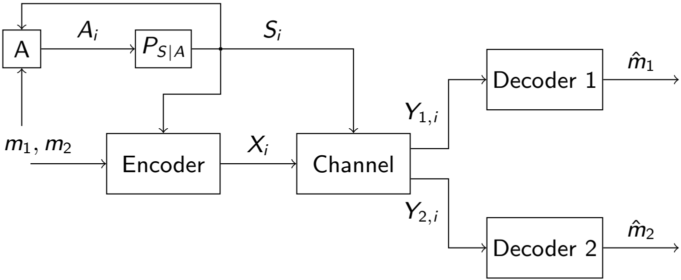


$$A_i = A_i(m_1, m_2, S^{i-1})$$

Main results

Action feedback

Q: What happens if the actions depend on past values of the states?



$$A_i = A_i(m_1, m_2, S^{i-1})$$

In single user channels, capacity is not increased (both cases, $X_i(m, S^i)$ or $X_i(m, S^n)$).

Main results

Action feedback

Theorem

The capacity region of the physically degraded broadcast channel with action-dependent states known causally at the encoder is not increased if the actions depend on past values of the states.

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Action feedback does increase the capacity region of the stochastically degraded broadcast channel with action dependent states.

Main results

Action feedback

Theorem

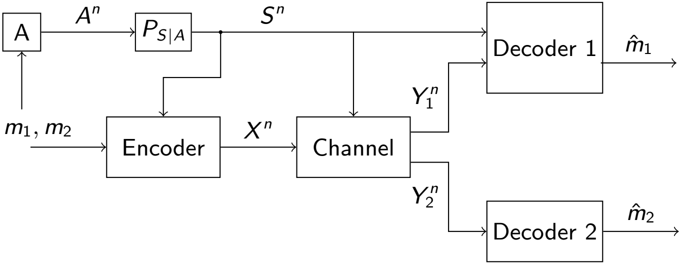
The capacity region of the physically degraded broadcast channel with action-dependent states known causally at the encoder is not increased if the actions depend on past values of the states.

Action feedback does increase the capacity region of the stochastically degraded broadcast channel with action dependent states.

Resembles the situation in BC without states (El Gamal 78, Dueck 80).

Main results

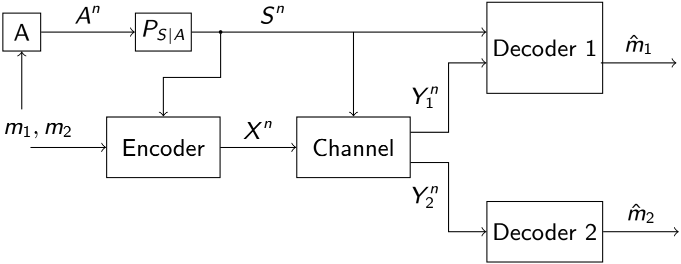
Non-causal SI, informed strong decoder



- Even without actions, the state-dependent degraded BC with non-causal SI is still an open problem.
- Solved for the case where the stronger user is informed.

Main results

Non-causal SI, informed strong decoder



An achievable region:

$$R_1 \leq I(X; Y_1|K, A)$$
$$R_2 \leq I(K, A; Y_2) - I(K; S|A)$$
$$\mathrm{E}\left[\Lambda(A, X)\right] \leq \lambda$$

$$P_A P_{S|A} P_{K|S,A} P_{X|S,K,A} P_{Y_1,Y_2|S,X}$$

Summary

- The degraded BC with action-dependent states and causal SI is solved.
- Future work: Non-causal setting. Good examples.