The Multiple Access Channel with Two Independent States each Known Causally to One Encoder

Amos Lapidoth and Yossef Steinberg
Outline
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- Problem Formulation: The MAC with strictly causal and causal independent SI
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- Background and related results:
  - The single user channel
  - Broadcast channels
  - MAC with common SI
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Problem Formulation

MAC with strictly causal side information (SI):

\[ P_{Y|S_1,S_2,X_1,X_2} \]

\[ P_{S_1,S_2} = P_{S_1} \cdot P_{S_2} \]
**Problem Formulation**

MAC with strictly causal side information (SI):

Two independent state sequences $S^n_1, S^n_2$ each known to one encoder in a strictly causal manner:

$$X_{1,i} = f_{1,i}(m_1, S^{-1}_1), \quad X_{2,i} = f_{2,i}(m_2, S^{-1}_2), \quad i = 1, \ldots, n$$
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Two independent state sequences $S_{1}^{n}$, $S_{2}^{n}$ each known to one encoder in a strictly causal manner:

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\[ (\hat{m}_{1}, \hat{m}_{2}) = g(Y^{n}) \]
Problem Formulation

MAC with strictly causal side information (SI):

\[ S_{1,i} \]

\[ m_1 \]

\[ X_{1,i} \rightarrow P_{Y|S_1,S_2,X_1,X_2} \]

\[ Y_i \rightarrow \hat{m}_1, \hat{m}_2 \]

\[ m_2 \]

\[ X_{2,i} \]

\[ S_{2,i} \]

Two independent state sequences \( S_{1}^{n}, S_{2}^{n} \) each known to one encoder in a strictly causal manner:

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X_{1,i} = f_{1,i}(m_1, S_{1,i}^{i-1}), \quad X_{2,i} = f_{2,i}(m_2, S_{2,i}^{i-1}), \quad i = 1, \ldots, n
\]

\[ (\hat{m}_1, \hat{m}_2) = g(Y^n) \]

Transmission is subject to input constraints \( \frac{1}{n} \sum_{i=1}^{n} \phi_k(X_{k,i}) \leq \Gamma_k, \quad k = 1, 2. \)
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Memoryless, time invariant channel and states \( P_{Y|S, X_1, X_2}, P_{S_1}, P_{S_2} \).
Problem Formulation

MAC with strictly causal side information (SI):

We are interested in $C^i_{s-c}$, the region of all achievable rate and cost pairs $(R_1, R_2, \Gamma_1, \Gamma_2)$. 
**Problem Formulation**

MAC with strictly causal side information (SI):

We are interested in $C_{s,c}^i$, the region of all achievable rate and cost pairs

$$(R_1, R_2, \Gamma_1, \Gamma_2).$$

$C_{s,c}^i(\Gamma_1, \Gamma_2)$ – the collection of all rate pairs $(R_1, R_2)$ such that

$$(R_1, R_2, \Gamma_1, \Gamma_2) \in C_{s,c}^i.$$
Problem Formulation

MAC with causal SI:

Two state sequences $S_1^n, S_2^n$, each known to one encoder in a causal manner:

$$X_{1,i} = f_{1,i}(m_1, S_1^i), \quad X_{2,i} = f_{2,i}(m_2, S_2^i), \quad i = 1, \ldots, n$$

$$(\hat{m}_1, \hat{m}_2) = g(Y^n)$$

Transmission is subject to input constraints $\frac{1}{n} \sum_{i=1}^{n} \phi_k(X_k,i) \leq \Gamma_k, \quad k = 1, 2$.

Memoryless, time invariant channel and state $P_{Y|S,X_1,X_2}, P_{S_1}, P_{S_2}$. 
Problem Formulation

MAC with causal SI:

\[
S_{1,i} \xrightarrow{m_1} \text{Encoder 1} \xrightarrow{X_{1,i}} P_{Y|S_1,S_2,X_1,X_2} \xrightarrow{Y_i} \text{Decoder} \xrightarrow{\hat{m}_1, \hat{m}_2} \\
S_{2,i} \xrightarrow{m_2} \text{Encoder 2} \xrightarrow{X_{2,i}} \]

We are interested in \( C_{\text{cau}} \), the region of all achievable rate and cost pairs

\[
(R_1, R_2, \Gamma_1, \Gamma_2). 
\]

\( C_{\text{cau}}(\Gamma_1, \Gamma_2) \) – the collection of all rate pairs \((R_1, R_2)\) such that

\[
(R_1, R_2, \Gamma_1, \Gamma_2) \in C_{\text{cau}}. 
\]
The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel
The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel

\[ nR - n\epsilon_n \leq I(M; Y^n) = \sum_{i=1}^{n} I(M; Y_i | Y^{i-1}) \]

\[ \leq \sum_{i=1}^{n} I(M, Y^{i-1}; Y_i) \]

\[ \leq \sum_{i=1}^{n} I(M, Y^{i-1}, X_i; Y_i) \]

\[ = \sum_{i=1}^{n} I(X_i; Y_i) \]

\[ \leq \max_{P_X} I(X; Y) = nC \]

where \( C \) is the capacity without SI.
The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel
  (a reminiscent of the situation in feedback capacity)
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- Transmission of the state (or compressed version thereof) to the other side is sub optimal: waste of precious rate, without increase in capacity.
The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel (a reminiscent of the situation in feedback capacity)
- Transmission of the state (or compressed version thereof) to the other side is sub optimal: waste of precious rate, without increase in capacity.

What about networks (BC, MAC)?
The broadcast channel with SC SI

- An example by Dueck (1980): A non degraded additive noise BC with feedback. The noise is common to the two channels.
The broadcast channel with SC SI

- An example by Dueck (1980): A non degraded additive noise BC with feedback. The noise is common to the two channels.

⇒ Equivalent to BC with strictly causal SI, where the state comprises the channel noise
The broadcast channel with SC SI

- An example by Dueck (1980): A non degraded additive noise BC with feedback.
  The noise is common to the two channels.
  - The encoder transmits the noise to the two users, uncompressed.
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  - Knowledge of the additive noise at the decoder facilitates decoding of the messages.
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  - Although precious rate is spent on transmitting the noise, the net effect is an increase in the capacity region.
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  - Yields gains in capacity also when only lossy transmission of the noise is possible.
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  - The encoder transmits the noise to the two users, uncompressed.
  - Knowledge of the additive noise at the decoder facilitates decoding of the messages.
  - Although precious rate is spent on transmitting the noise, the net effect is an increase in the capacity region.
  - Yields gains in capacity also when only lossy transmission of the noise is possible.

- In the MAC: If the state is known to both users, they can cooperate in transmitting the noise (state) to the decoder. This strategy enlarges the capacity region of the MAC [Lapidoth & Steinberg, IZS 2010].
MAC with SC common SI

[Lapidoth & Steinberg, IZS2010]:

\[
\begin{align*}
&\text{Encoder 1} \quad \begin{array}{c}
\text{Encoder 2} \\
\downarrow \quad m_1 \\
X_{1,i} \quad Y_i \quad \hat{m}_1, \hat{m}_2 \\
\downarrow \quad \uparrow \quad S_i \\
S_{i-1} \quad P_{Y|S,X_1,X_2} \\
\downarrow \quad \downarrow \quad m_2 \\
X_{2,i} \quad \hat{m}_1, \hat{m}_2 \\
\end{array}
\end{align*}
\]
MAC with SC common SI

\( \mathcal{R}_{\text{S-c}} \) - the CH of all \((R_1, R_2, \Gamma_1, \Gamma_2)\) satisfying

\[
\begin{align*}
R_1 & \leq I(X_1; Y|X_2, U, V) \\
R_2 & \leq I(X_2; Y|X_1, U, V) \\
R_1 + R_2 & \leq I(X_1, X_2; Y|U, V) \\
R_1 + R_2 & \leq I(X_1, X_2, V; Y) - I(V; S) \\
\Gamma_k & \geq \mathbb{E}[\phi_k(X_k)], \quad k = 1, 2
\end{align*}
\]

for some joint distribution

\[
P_{U,V,X_1,x_2,S,Y} = P_S P_{X_1|U} P_{X_2|U} P_U P_V|S P_Y|S,x_1,x_2.
\]
MAC with SC common SI

\( \mathcal{R}_{s-c}^{\text{common}} \) - the CH of all \((R_1, R_2, \Gamma_1, \Gamma_2)\) satisfying

\[
\begin{align*}
R_1 & \leq I(X_1; Y|X_2, U, V) \\
R_2 & \leq I(X_2; Y|X_1, U, V) \\
R_1 + R_2 & \leq I(X_1, X_2; Y|U, V) \\
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\]

\[
X_1 - U - X_2 \\
(X_1, U, X_2) \perp (V, S)
\]
MAC with SC common SI

$\mathcal{R}^{\text{common}}_{S-c}$ - the CH of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

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V - S - Y
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MAC with SC common SI

$R_{s-c}^\text{common}$ - the CH of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

\[
R_1 \leq I(X_1; Y|X_2, U, V)
\]

\[
R_2 \leq I(X_2; Y|X_1, U, V)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|U, V)
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R_1 + R_2 \leq I(X_1, X_2, V; Y) - I(V; S)
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\Gamma_k \geq \mathbb{E}[\phi_k(X_k)], \quad k = 1, 2
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Theorem 1 [L&S, IZS 2010]

For the MAC with strictly causal SI commonly known by the two encoders, $R_{s-c}^\text{common}$ is achievable.
MAC with SC common SI

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Theorem 1 [L&S, IZS 2010]

For the MAC with strictly causal SI commonly known by the two encoders, \( \mathcal{R}_{s-c}^{\text{common}} \) is achievable.

Observation: \( \mathcal{R}_{s-c}^{\text{common}} \) can be strictly larger than the capacity region without SI.
Background

We can write $\mathcal{R}_{s,c}^{\text{common}}$ as

\begin{align*}
R_1 & \leq I(X_1; Y|X_2, U, V) \\
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R_0 + R_1 + R_2 & \leq I(X_1, X_2; Y|V) \\
R_0 & \geq I(V; S) - I(V; Y). \\
\Gamma_k & \geq \mathbb{E}[\phi_k(X_k)], \quad k = 1, 2
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$$R_0 \geq I(V; S) - I(V; Y).$$
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Based on MAC with common messages + block Markov scheme:
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Based on MAC with common messages + block Markov scheme:

- The state sequence $S^n$ is compressed by a Wyner-Ziv scheme, with coding random variable $V$, and decoder side information $Y^n$. 

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- The state sequence $S^n$ is compressed by a Wyner-Ziv scheme, with coding random variable $V$, and decoder side information $Y^n$.
- The compressed state is transmitted to the decoder in the next transmission block as a common message, together with the independent messages $m_1, m_2$. 

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Based on MAC with common messages + block Markov scheme:

- The state sequence $S^n$ is compressed by a Wyner-Ziv scheme, with coding random variable $V$, and decoder side information $Y^n$.
- The compressed state is transmitted to the decoder in the next transmission block as a common message, together with the independent messages $m_1, m_2$.
- Cooperation is possible, since the state is common.
MAC with independent SI streams

Back to our problem:

\[ P_{S_1, S_2} = P_{S_1} \cdot P_{S_2} \]
**MAC with independent SI streams**

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The two encoders cannot establish cooperation of any kind
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Joint transmission of the states is not possible.
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- Each of the encoders is working alone – like in the single user channel.
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  - In this setup, is SC SI beneficial at all?
Back to our problem:

- The two encoders cannot establish cooperation of any kind. Joint transmission of the states is not possible.
- Each of the encoders is working alone – like in the single user channel.
  - In this setup, is SC SI beneficial at all?
  - If it is beneficial, is it a good idea to compress and transmit the states to the other side?
Main result

Let $\mathcal{R}_{\text{sc}}^i$ be the convex hull of the collection of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

$$0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)$$

$$0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)$$

$$\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2$$

for some $(V_1, V_2, S_1, S_2, X_1, X_2, Y)$ with joint distribution

$$P_{V_1|S_1} P_{V_2|S_2} P_{S_1} P_{S_2} P_{X_1} P_{X_2} P_{Y|S_1, S_2, X_1, X_2}.$$
Main result

Let $\mathcal{R}_{sc}^i$ be the convex hull of the collection of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

\[
0 \leq R_1 \leq I(X_1; Y | X_2, V_1, V_2) - I(V_1; S_1 | Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y | X_1, V_1, V_2) - I(V_2; S_2 | Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y | V_1, V_2) - I(V_1, V_2; S_1, S_2 | Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

\[
V_1 - S_1 - (V_2, Y, S_2)
\]

\[
V_2 - S_2 - (V_1, Y, S_1)
\]

\[
(V_1, V_2) - (S_1, S_2) - Y
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Main result

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\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

\[
V_1 - S_1 - (V_2, Y, S_2)
\]

\[
V_2 - S_2 - (V_1, Y, S_1)
\]

\[
(V_1, V_2) - (S_1, S_2) - Y
\]

$X_1, X_2$ are independent of each other and of the quadruple $(V_1, V_2, S_1, S_2)$. 
Main result

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\[
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\]
\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]
\[
\Gamma_k \geq \mathbb{E} \phi_k(X_k), \quad k = 1, 2
\]

\[
V_1 - S_1 - (V_2, Y, S_2)
\]
\[
V_2 - S_2 - (V_1, Y, S_1)
\]
\[
(V_1, V_2) - (S_1, S_2) - Y
\]

$X_1, X_2$ are independent of each other and of the quadruple $(V_1, V_2, S_1, S_2)$.

\[
(V_1, S_1) \perp (V_2, S_2)
\]
Main result

\( \mathcal{R}_{sc}^i \) - the convex hull of the collection of all \((R_1, R_2, \Gamma_1, \Gamma_2)\) satisfying

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \\
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \\
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \\
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

Theorem 2 (Strictly-Causal, independent SI streams)

\( \mathcal{R}_{sc}^i \subseteq C_{sc}^i \)
Main result

\[ 0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]
\[ 0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]
\[ R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]
\[ \Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2 \]
Main result

\[ 0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]
\[ 0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]
\[ R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]
\[ \Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2 \]

A block Markov scheme:
Main result

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

A block Markov scheme:

- The state sequences $S_1^n, S_2^n$ are compressed by a distributed Wyner-Ziv scheme, with coding random variable $V_1, V_2$ and decoder side information $Y^n$. 
Main result

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E} \phi_k(X_k), \quad k = 1, 2
\]

A block Markov scheme:

- The state sequences \( S_1^n, S_2^n \) are compressed by a *distributed* Wyner-Ziv scheme, with coding random variable \( V_1, V_2 \) and decoder side information \( Y^n \).

\[
(V_1, V_2) - (S_1, S_2) - Y
\]
Main result

\[ 0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]
\[ 0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]
\[ R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]
\[ \Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2 \]

A block Markov scheme:
- The state sequences \( S_1^n, S_2^n \) are compressed by a distributed Wyner-Ziv scheme, with coding random variable \( V_1, V_2 \) and decoder side information \( Y^n \).

\[ (V_1, V_2) - (S_1, S_2) - Y \]

- The compressed states are transmitted to the decoder in the next transmission block as independent codewords, together with the independent messages \( m_1, m_2 \).
Main result

\[ 0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]
\[ 0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]
\[ R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]
\[ \Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2 \]

A block Markov scheme:

- The state sequences \( S_1^n, S_2^n \) are compressed by a distributed Wyner-Ziv scheme, with coding random variable \( V_1, V_2 \) and decoder side information \( Y^n \).

\[ (V_1, V_2) - (S_1, S_2) - Y \]

- The compressed states are transmitted to the decoder in the next transmission block as independent codewords, together with the independent messages \( m_1, m_2 \).

\[ X_1 \perp X_2, \quad \text{independent of } (V_1, V_2, S_1, S_2). \]
Main result

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

A block Markov scheme:

- The state sequences $S^n_1$, $S^n_2$ are compressed by a \textit{distributed} Wyner-Ziv scheme, with coding random variable $V_1$, $V_2$ and decoder side information $Y^n$.

\[
(V_1, V_2) - (S_1, S_2) - Y
\]

- The compressed states are transmitted to the decoder in the \textit{next transmission block} as \textit{independent codewords}, together with the independent messages $m_1$, $m_2$.

\[
X_1 \perp X_2, \quad \text{independent of} \quad (V_1, V_2, S_1, S_2).
\]

- The two codes are decoupled.
Partial characterizations

Two propositions – about the sum rate, and about the asymmetric case.
Partial characterizations

Two propositions – about the sum rate, and about the asymmetric case.

Proposition 1  \textit{Strictly-causal independent SI does not increase the sum-rate capacity}:

\[ C^i_{\Sigma, s-c}(\Gamma_1, \Gamma_2) = \max I(X_1, X_2; Y), \]

\textit{where the maximum is over all product distributions }\textit{P}_{X_1} \textit{P}_{X_2} \textit{satisfying the input constraints}

\[ \mathbb{E}\phi_k(X_k) \leq \Gamma_k, \quad k = 1, 2. \]
Partial characterizations

The asymmetric case:

**Proposition 2** Let $S_2$ be deterministic. Then the maximal rate of User 1 with strictly causal SI is equal to its single user capacity without SI

$$\max \left\{ R_1 : (R_1, 0) \in C_{s-c}^i(\Gamma_1, \Gamma_2) \right\} = \max I(X_1; Y|X_2),$$

where the maximum in the right hand side is over all $P_{X_1}P_{X_2}$ satisfying the input constraints

$$\mathbb{E}\phi_k(X_k) \leq \Gamma_k, \quad k = 1, 2.$$
Example

The Gaussian MAC where the state $S_1$ comprises the channel noise, and $S_2$ is null:

$$Y = X_1 + X_2 + S_1, \quad S_1 \sim \mathcal{N}(0, \sigma_{s_1}^2)$$

$$\mathbb{E}[X_1^2] \leq \Gamma_1, \quad \mathbb{E}[X_2^2] \leq \Gamma_2.$$
Example

The Gaussian MAC where the state $S_1$ comprises the channel noise, and $S_2$ is null:

$$Y = X_1 + X_2 + S_1, \quad S_1 \sim \mathcal{N}(0, \sigma_{s_1}^2)$$

$$E \left[ X_1^2 \right] \leq \Gamma_1, \quad E \left[ X_2^2 \right] \leq \Gamma_2.$$

$C_{s-c}^i(\Gamma_1, \Gamma_2)$ is the collection of all rate-pairs $(R_1, R_2)$ satisfying

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1}{\sigma_{s_1}^2} \right)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1 + \Gamma_2}{\sigma_{s_1}^2} \right).$$
Example

The Gaussian MAC where the state $S_1$ comprises the channel noise, and $S_2$ is null:

$$Y = X_1 + X_2 + S_1, \quad S_1 \sim \mathcal{N}(0, \sigma_{s1}^2)$$

$$\mathbb{E}[X_1^2] \leq \Gamma_1, \quad \mathbb{E}[X_2^2] \leq \Gamma_2.$$

$C_{sc}^{i}(\Gamma_1, \Gamma_2)$ is the collection of all rate-pairs $(R_1, R_2)$ satisfying

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1}{\sigma_{s1}^2} \right)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1 + \Gamma_2}{\sigma_{s1}^2} \right).$$

Proof:

Direct part: good choice of random variables in $R_{sc}^{i}$. 
Example

The Gaussian MAC where the state $S_1$ comprises the channel noise, and $S_2$ is null:

$$Y = X_1 + X_2 + S_1, \quad S_1 \sim \mathcal{N}(0, \sigma_{s_1}^2)$$

$$\mathbb{E}[X_1^2] \leq \Gamma_1, \quad \mathbb{E}[X_2^2] \leq \Gamma_2.$$

$C^i_{s-c}(\Gamma_1, \Gamma_2)$ is the collection of all rate-pairs $(R_1, R_2)$ satisfying

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1}{\sigma_{s_1}^2} \right)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\Gamma_1 + \Gamma_2}{\sigma_{s_1}^2} \right).$$

Proof:

Direct part: good choice of random variables in $R^i_{sc}$.

Converse: use Propositions 1 and 2.
Example
Example

- User 1 knows the noise in a strictly causal manner, but cannot utilize it to increase his own rate.
Example

- User 1 knows the noise in a strictly causal manner, but cannot utilize it to increase his own rate.
- He can use it to increase the rate of User 2.
MAC with causal SI

The region we had for the strictly causal case is still achievable

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]
MAC with causal SI

The region we had for the strictly causal case is still achievable

\[ 0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]
\[ 0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]
\[ R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]
\[ \Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2 \]

with the Markov conditions

\[ V_1 - S_1 - (V_2, Y, S_2) \]
\[ V_2 - S_2 - (V_1, Y, S_1) \]
\[ (V_1, V_2) - (S_1, S_2) - Y \]
\[ X_1 \perp X_2, \quad (X_1, X_2) \perp (V_1, V_2, S_1, S_2). \]
MAC with causal SI

The region we had for the strictly causal case is still achievable

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E} \phi_k(X_k), \quad k = 1, 2
\]

with the Markov conditions

\[
V_1 - S_1 - (V_2, Y, S_2)
\]

\[
V_2 - S_2 - (V_1, Y, S_1)
\]

\[
(V_1, V_2) - (S_1, S_2) - Y
\]

\[
X_1 \perp X_2, \quad (X_1, X_2) \perp (V_1, V_2, S_1, S_2).
\]

But now, \(X_1, X_2\) can depend on \(S\).
**MAC with causal SI**

The region we had for the strictly causal case is still achievable

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2)
\]

\[
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)
\]

\[
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

with the Markov conditions

\[
V_1 - S_1 - (V_2, Y, S_2)
\]

\[
V_2 - S_2 - (V_1, Y, S_1)
\]

\[
(V_1, V_2) - (S_1, S_2) - Y
\]

\[
X_1 \perp X_2, \quad (X_1, X_2) \perp (V_1, V_2, S_1, S_2).
\]

But now, \(X_1, X_2\) can depend on \(S\).

⇒ Use Shannon strategies on top of our block Markov scheme.
MAC with causal SI

The region we had for the strictly causal case is still achievable

\[
0 \leq R_1 \leq I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \\
0 \leq R_2 \leq I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \\
R_1 + R_2 \leq I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \\
\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2
\]

with the Markov conditions

\[
V_1 - S_1 - (V_2, Y, S_2) \\
V_2 - S_2 - (V_1, Y, S_1) \\
(V_1, V_2) - (S_1, S_2) - Y \\
X_1 \perp X_2, \quad (X_1, X_2) \perp (V_1, V_2, S_1, S_2).
\]

But now, \(X_1, X_2\) can depend on \(S\).

Replace \((X_1, X_2)\) by \((U_1, U_2)\) independent of \((S_1, S_2)\), and let

\[
P_{X_1|U_1, S_1}, \quad P_{X_2|U_2, S_2}
\]
Main result

$R_{cau}^i$ - the CH of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

$$0 \leq R_1 \leq I(U_1; Y|U_2, V_1, V_2) - I(V_1; S_1|Y, V_2)$$

$$0 \leq R_2 \leq I(U_2; Y|U_1, V_1, V_2) - I(V_2; S_2|Y, V_1)$$

$$R_1 + R_2 \leq I(U_1, U_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y)$$

$$\Gamma_k \geq \mathbb{E}\phi_k(X_k), \quad k = 1, 2$$

for some $(V_1, V_2, U_1, U_2, S_1, S_2, X_1, X_2, Y)$ with joint distribution

$$P_{V_1|S_1} P_{V_2|S_2} P_{U_1} P_{U_2} P_{S_1} P_{S_2} P_{X_1|U_1, S_1} P_{X_2|U_2, S_2} P_Y|S_1, S_2, X_1, X_2.$$
Main result

\[ \mathcal{R}_{\text{cau}}^i - \text{the CH of all } (R_1, R_2, \Gamma_1, \Gamma_2) \text{ satisfying} \]

\[ 0 \leq R_1 \leq I(U_1; Y|U_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \]

\[ 0 \leq R_2 \leq I(U_2; Y|U_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \]

\[ R_1 + R_2 \leq I(U_1, U_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \]

\[ \Gamma_k \geq \mathbb{E} \phi_k(X_k), \quad k = 1, 2 \]

for some \((V_1, V_2, U_1, U_2, S_1, S_2, X_1, X_2, Y)\) with joint distribution

\[ P_{V_1|S_1} P_{V_2|S_2} P_{U_1} P_{U_2} P_{S_1} P_{S_2} P_{X_1|U_1, S_1} P_{X_2|U_2, S_2} P_{Y|S_1, S_2, X_1, X_2}. \]

Theorem 3 (Causal, independent SI streams)

\[ \mathcal{R}_{\text{cau}}^i \subseteq \mathcal{C}_{\text{cau}}^i \]
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state.
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all $(R_1, R_2)$ satisfying

\[
R_1 \leq I(T_1; Y|T_2, Q)
\]

\[
R_2 \leq I(T_2; Y|T_1, Q)
\]

\[
R_1 + R_2 \leq I(T_1, T_2; Y|Q)
\]

for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_Y|T_1, T_2$. 
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all \((R_1, R_2)\) satisfying

\[
R_1 \leq I(T_1; Y|T_2, Q)
\]

\[
R_2 \leq I(T_2; Y|T_1, Q)
\]

\[
R_1 + R_2 \leq I(T_1, T_2; Y|Q)
\]

for some joint distribution \(P_Q P_{T_1}|Q P_{T_2}|Q P_Y|T_1, T_2\). Here \(T_k, k = 1, 2\) are random Shannon strategies:

\[
T_k \in \mathcal{T}_k, \quad \text{the set of mappings} \quad t_k : S_k \rightarrow X_k
\]
**The naïve approach**

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all \((R_1, R_2)\) satisfying

\[
\begin{align*}
R_1 & \leq I(T_1; Y|T_2, Q) \\
R_2 & \leq I(T_2; Y|T_1, Q) \\
R_1 + R_2 & \leq I(T_1, T_2; Y|Q)
\end{align*}
\]

for some joint distribution \(P_Q P_{T_1|Q} P_{T_2|Q} P_Y|T_1, T_2\). Here \(T_k, k = 1, 2\) are random Shannon strategies:

\[
T_k \in \mathcal{T}_k, \quad \text{the set of mappings} \quad t_k : S_k \rightarrow \mathcal{X}_k
\]

\(Q\) is a time sharing random variable,
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all \((R_1, R_2)\) satisfying

\[
R_1 \leq I(T_1; Y|T_2, Q) \\
R_2 \leq I(T_2; Y|T_1, Q) \\
R_1 + R_2 \leq I(T_1, T_2; Y|Q)
\]

for some joint distribution \(P_Q P_{T_1|Q} P_{T_2|Q} P_Y|T_1, T_2\). Here

\(T_k, k = 1, 2\) are random Shannon strategies:

\[T_k \in \mathcal{T}_k, \quad \text{the set of mappings} \quad t_k : S_k \to X_k\]

\(Q\) is a time sharing random variable, and

\[
P_{Y|T_1, T_2}(y|t_1, t_2) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} P_{S_1}(s_1) P_{S_2}(s_2) \cdot P_{Y|S_1, S_2, X_1, X_2}(y|s_1, s_2, t_1(s_1), t_2(s_2)).
\]
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all \((R_1, R_2)\) satisfying

\[
R_1 \leq I(T_1; Y|T_2, Q) \\
R_2 \leq I(T_2; Y|T_1, Q) \\
R_1 + R_2 \leq I(T_1, T_2; Y|Q)
\]

for some joint distribution \(P_Q P_{T_1|Q} P_{T_2|Q} P_Y|T_{1,2}\).

We denote this region as \(\mathcal{R}^{\text{naïve}}\).
The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all \((R_1, R_2)\) satisfying

\[
R_1 \leq I(T_1; Y | T_2, Q)
\]

\[
R_2 \leq I(T_2; Y | T_1, Q)
\]

\[
R_1 + R_2 \leq I(T_1, T_2; Y | Q)
\]

for some joint distribution \(P_Q P_{T_1} | Q P_{T_2} | Q P_Y | T_1, T_2\).

We denote this region as \(\mathcal{R}^{\text{naïve}}\).

\(\mathcal{R}^{\text{naïve}}\) contains the region suggested in [S.A. Jafar, Dec 2006].
The naïve approach

- $R_{cau}^i$ contains the region of the naïve approach, since we can always choose deterministic $(V_1, V_2)$.

- In some cases, the inclusion is strict.
Example

The asymmetric state-dependent MAC consisting of two single user channels:

\[ \mathcal{X}_1 = \{0, 1\}, \quad \mathcal{X}_2 = \{0, 1, 2, 3\}, \quad \mathcal{Y} = \mathcal{Y}_1 \times \mathcal{Y}_2 \]

\[ \mathcal{Y}_1 = \{0, 1\}, \quad \mathcal{Y}_2 = \{0, 1, 2, 3\}. \]
Example

The asymmetric state-dependent MAC consisting of two single user channels:

\[ X_1 = \{0, 1\}, \quad X_2 = \{0, 1, 2, 3\}, \quad Y = Y_1 \times Y_2 \]

\[ Y_1 = \{0, 1\}, \quad Y_2 = \{0, 1, 2, 3\}. \]

The channel is defined as

\[ Y_1 = X_1 \]
\[ Y_2 = X_2 \oplus S_1, \]
Example

The asymmetric state-dependent MAC consisting of two single user channels:

\[ X_1 = \{0, 1\}, \quad X_2 = \{0, 1, 2, 3\}, \quad Y = Y_1 \times Y_2 \]

\[ Y_1 = \{0, 1\}, \quad Y_2 = \{0, 1, 2, 3\}. \]

The channel is defined as

\[ Y_1 = X_1 \]
\[ Y_2 = X_2 \oplus S_1, \]

where

\[ S_1 = \{0, 1, 2, 3\}, \quad P_{S_1} = (1 - p, p/3, p/3, p/3), \quad H(S_1) < 1. \]
Example

\[ Y_1 = X_1, \quad \text{binary} \]

\[ Y_2 = X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1. \]
Example

\[ Y_1 = X_1, \quad \text{binary} \]

\[ Y_2 = X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1. \]

What is the maximal transmission rate of user 2 under each of the schemes?
Example

\[ Y_1 = X_1, \quad \text{binary} \]

\[ Y_2 = X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1. \]

What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields \( R_{2,\text{max}}^{(bm)} = 2. \)
Example

\[ Y_1 = X_1, \quad \text{binary} \]
\[ Y_2 = X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1. \]

What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields \( R_{2,\text{max}}^{(bm)} = 2 \).

\[ \text{Achievability - by proper choice of random variables in } R_{\text{cau}}. \]
Example

\[
\begin{align*}
Y_1 &= X_1, \quad \text{binary} \\
Y_2 &= X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1.
\end{align*}
\]

What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields \( R_{2,\text{max}}^{\text{bm}} = 2 \).

Achievability - by proper choice of random variables in \( \mathcal{R}_{\text{cau}}^1 \).

This is tight, since \(|\mathcal{X}_2| = 4\).
Example

\[ Y_1 = X_1, \quad \text{binary} \]
\[ Y_2 = X_2 \oplus S_1, \quad \text{quaternary with } H(S_1) < 1. \]

What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields \( R_{2,\text{max}}^{\text{(bm)}} = 2 \).

  Achievability - by proper choice of random variables in \( R_{\text{cau}}^i \).

  This is tight, since \(|\mathcal{X}_2| = 4\).

- It can be shown that \( R_{2,\text{max}}^{\text{(naïve)}} < 2 \).
Summary

- Derived achievable region for the MAC with two independent strictly causal SI streams, based on block Markov encoding of the state.
- Although cooperation between the users is impossible in this setup, strictly causal SI enlarges the capacity region of the MAC.
- Extended the results to causal SI
- The new region for causal SI is strictly better than the region obtained by the naïve approach, which utilizes only Shannon strategies without block-Markov coding.
Thank You!