Channels with Cooperation Links that May Be Absent

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***Abstract*—It is well known that cooperation between users in a communication network can lead to significant performance gains. A common assumption in past works is that all the users are aware of the resources available for cooperation, and know exactly to what extent these resources can be used. In this work a family of models is suggested where the cooperation links may or may not be present. Coding schemes are devised that exploit the cooperation links if they are present, and can still operate (although at reduced rates) if cooperation is not possible.**

***Index Terms*—Broadcast channel, conferencing decoders, cooperation, cribbing, multiple access channel.**

I. Introduction

Communication techniques that employ cooperation between users in a network have been an extensive area of research in recent years. The interest in such schemes stems from the potential increase in the network performance. The employment of cooperative schemes require the use of system resources - bandwidth, time slots, energy, etc - that should be allocated for the cooperation to take place. Due to the dynamic nature of modern, wireless ad-hoc communication systems, the availability of these resources is not guaranteed a priori, and the coding schemes are required to work also in the absence of the cooperation links, although possibly achieving lower communication rates.

In this work we study channels with cooperation links that may or may not be present. We focus on two cases - the physically degraded broadcast channel (BC) with conferencing decoders, and the multiple access channel (MAC) with cribbing encoders. The BC with conferencing decoders was first studied by Dabora and Servetto [2], [3], and independently by Liang and Veeravalli [6], [7], who studied also the more general setting of relay-broadcast channels (RBC). In the model of Dabora and Servetto, a two-users BC is considered, where the decoders can exchange information via noiseless communication links of limited capacities $C_{1,2}$ and $C_{2,1}$. When the broadcast channel is physically degraded, information sent from the weaker (degraded) user to the stronger is redundant, and only the capacity of the link from the stronger user to the weaker (say $C_{1,2}$) increases the communication rates. For this case, Dabora and Servetto characterized the capacity region. Their result coincides with the results of Liang and Veeravalli when the relay link of [6] is replaced with a constant rate bit pipe.

The MAC with cribbing encoders was introduced by Willems and Van Der Meulen in [9]. Here there is no dedicated communication link that can be used explicitly for cooperation. Instead, one of the encoders can crib, or listen, to the channel input of the other user. This model describes a situation in which users in a cellular system are located physically close to each other, enabling part of them to listen to the transmission of the others with high reliability - i.e., the channel between the transmitters that are located in close vicinity is almost noiseless. Willems and Van Der Meulen considered in [9] all consistent scenarios of cribbing (strictly causal, causal, non-causal, and symmetric or asymmetric), and characterized the capacity region of these models.

In the next sections, we propose and study extensions of the two models described above, when the cooperation links ($C_{1,2}$ of the physically degraded BC, and the cribbing link of the MAC) may or may not be present. Achievable rates are proposed for the MAC models. For the physically degraded BC, the results are conclusive.

It should be noted that multi user communication systems with uncertainty in part of the network links have been extensively studied in the literature - see, e.g., [8] and [5], and references therein. A comprehensive survey of these works is not presented here due to space limitations. The models suggested here, of the BC and MAC with uncertainty in the cooperation links, have not been studied before.

II. The Physically Degraded Broadcast Channel with Cooperating Decoders

Let $\mathcal{X}$, $\mathcal{Y}_1$, $\mathcal{Y}_2$ be finite sets. A broadcast channel (BC) $(\mathcal{X}, \mathcal{Y}_1, \mathcal{Y}_2, P_{Y_1,Y_2|X})$ is a channel with input alphabet $\mathcal{X}$, two output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$, and a transition probability $P_{Y_1,Y_2|X}$ from $\mathcal{X}$ to $\mathcal{Y}_1 \times \mathcal{Y}_2$. The BC is said to be physically degraded if for any input distribution P_X, the Markov chain $X \multimap Y_1 \multimap Y_2$ holds, i.e.,

$$P_{X,Y_1,Y_2} = P_X P_{Y_1,Y_2|X} = P_X P_{Y_1|X} P_{Y_2|Y_1}. \tag{1}$$

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We will refer to Y_1 (resp. Y_2) as the stronger (resp. weaker, or degraded) user. We assume throughout that the channel is memoryless and that no feedback is present, implying that the transition probability of n-sequences is given by

$$P_{Y_1,Y_2|X}(y_1^n, y_2^n|x^n) = \prod_{i=1}^{n} P_{Y_1,Y_2|X}(y_{1,i}, y_{2,i}|x_i) \quad (2)$$

Fix the transmission length, n, and an integer $\nu_{1,2}$. Let $\mathcal{N}_{1,2} = \{1, 2, \ldots, \nu_{1,2}\}$ be the index set of the conference message. Denote the sets of messages by $\mathcal{N}_k = \{1, 2, \ldots, \nu_k\}$, $k = 1, 2$, and $\mathcal{N}_2' = \{1, 2, \ldots, \nu_2'\}$ where ν_1, ν_2 and ν_2' are integers. A code for the BC with unreliable conference link, that may or may not be present, operates as follows. Three messages M_1, M_2, and M_2' are drawn uniformly and independently from the sets $\mathcal{N}_1$, $\mathcal{N}_2$, and $\mathcal{N}_2'$, respectively. The encoder maps this triplet to a channel input sequence, $\boldsymbol{x}(M_1, M_2, M_2')$. At the channel output, Decoder k has the output sequence Y_k^n, $k = 1, 2$, at hand. Decoder 1 (resp. Decoder 2) is required to decode the message M_1 (resp. M_2), whether or not the conference link is present. If the conference link is present, Decoder 1 sends a message $c \in \mathcal{N}_{1,2}$ to Decoder 2, based on the output sequence Y_1^n. I.e., $c = c(Y_1^n)$. Finally, Decoder 2 decodes M_2' based on his output Y_2^n and the conference message $c(Y_1^n)$.

Remark 1: Observe that only Decoder 2 benefits when the conference link is present. Indeed, since there is only a link from Decoder 1 to Decoder 2, whatever Decoder 1 can do with the link, he can also do without it. Therefore the rate to User 1 is independent of whether the link is present or not. Only User 2 can benefit from its existence, and thus there are two sets of messages intended to User 2 - $\mathcal{N}_2$ and $\mathcal{N}_2'$.
Following is a more formal description.

Definition 1: An $(n, \nu_1, \nu_2, \nu_2', \nu_{1,2}, \epsilon)$ code for the BC $P_{Y_1,Y_2|X}$ with an unreliable conference link is an encoder mapping

$$f : \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{M}_2' \to \mathcal{X}^n,$$

a conference mapping

$$h : \mathcal{Y}_1^n \to \mathcal{N}_{1,2}$$

and three decoding maps:

$$g_1 : \mathcal{Y}_1^n \to \mathcal{N}_1 \quad (3a)$$
$$g_2 : \mathcal{Y}_2^n \to \mathcal{N}_2 \quad (3b)$$
$$g_2' : \mathcal{Y}_2^n \times \mathcal{N}_{1,2} \to \mathcal{N}_2' \quad (3c)$$

such that the average probabilities of error P_e and P_e' do not exceed ϵ. Here

$$P_e = \frac{1}{\nu_1 \nu_2 \nu_2'} \sum_{m_1, m_2, m_2'} P_{Y_1,Y_2|X}(S_e|f(m_1, m_2, m_2')) \quad (4a)$$
$$P_e' = \frac{1}{\nu_1 \nu_2 \nu_2'} \sum_{m_1, m_2, m_2'} P_{Y_1,Y_2|X}(S_e'|f(m_1, m_2, m_2')) \quad (4b)$$

where the sets S_e and S_e' are defined as

$$S_e(m_1, m_2) = \{(\boldsymbol{y}_1, \boldsymbol{y}_2) : g_1(\boldsymbol{y}_1) \neq m_1 \text{ or } g_2(\boldsymbol{y}_2) \neq m_2\}$$

$$S_e'(m_1, m_2, m_2') = S_e(m_1, m_2) \cup \{(\boldsymbol{y_1}, \boldsymbol{y}_2) : g_2'(\boldsymbol{y}_2, h(\boldsymbol{y}_1)) \neq m_2'\},$$

and for notational convenience, the dependence of S_e and S_e' on the messages is dropped in (4).
The conference rate $C_{1,2}$ and the communications rates (R_1, R_2, R_2') are defined as usual:

$$C_{1,2} = \frac{\log \nu_{1,2}}{n}, \quad R_k = \frac{\log \nu_k}{n}, k = 1, 2, \quad R_2' = \frac{\log \nu_2'}{n}.$$

Remark 2: The interpretation of the rates is as follows: $C_{1,2}$ is the conference rate in case that it is present. The rate R_k is intended to User k, $k = 1, 2$, to be decoded whether or not the conference is present. The rate R_2' is intended to User 2 and is the extra rate gained if the conference link is present.

A rate quadruple $(R_1, R_2, R_2', C_{1,2})$ is said to be achievable with unreliable conference if for any $\epsilon > 0$, $\gamma > 0$, and sufficiently large n there exists an $(n, 2^{n(R_1-\gamma)}, 2^{n(R_2-\gamma)}, 2^{n(R_2'-\gamma)}, 2^{n(C_{1,2}+\gamma)}, \epsilon)$ code for the BC with unreliable conference link. The capacity region is the closure of the set of all achievable quadruples $(R_1, R_2, R_2', C_{1,2})$ and is denoted by $\mathcal{C}$. For a given conference rate $C_{1,2}$, $\mathcal{C}(C_{1,2})$ stands for the section of $\mathcal{C}$ at $C_{1,2}$. Our interest is to characterize $\mathcal{C}(C_{1,2})$.

Let $\mathcal{R}(C_{1,2})$ be the set of all rate triples (R_1, R_2, R_2') satisfying

$$R_2 \leq I(U; Y_2) \quad (5a)$$
$$R_2' \leq \min\{I(V; Y_2|U) + C_{1,2}, I(V, Y_1|U)\} \quad (5b)$$
$$R_1 \leq I(X; Y_1|U, V) \quad (5c)$$

for some joint distribution of the form

$$P_{U,V,X,Y_1,Y_2} = P_{U,V} P_{X|U,V} P_{Y_1,Y_2|X}. \quad (5d)$$

Our main result on the physical degraded BC with unreliable conference is the following

Theorem 1: For any physically degraded BC with unreliable conference of rate $C_{1,2}$,

$$\mathcal{C}(C_{1,2}) = \mathcal{R}(C_{1,2}).$$

Due to lack of space, a detailed proof is omitted. Sketch of the proof is given in Section IV.

It is interesting to examine the region $\mathcal{R}(C_{1,2})$ in extreme cases. Assume that $C_{1,2} = 0$, that is, the case where even if the conference link is present, its rate is 0. Due to (5d) the Markov condition $(U, V) \multimap Y_1 \multimap Y_2$ holds, implying also that $V \multimap (U, Y_1) \multimap Y_2$ holds. Therefore, when $C_{1,2} = 0$, the bounds in (5) reduce to

$$R_2 \leq I(U; Y_2) \quad (6a)$$
$$R_2' \leq I(V; Y_2|U) \quad (6b)$$
$$R_1 \leq I(X; Y_1|U, V). \quad (6c)$$

The total rate to User 2 is $R_2 + R_2'$. It is easy to verify that after optimization over (U, V), the rates guaranteed by (6) coincide with the capacity region of the degraded BC. Another case of interest is when $R_2 = 0$. Here User 2 will not get any rate if the conference link is absent. Choosing U to be a null random variable, the region of rates (R_1, R_2') guaranteed by (5) coincides with the result in [3, Theorem 1].

III. The Multiple Access Channel with Cribbing Encoders

A multiple access channel (MAC) is a quadruple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, P_{Y|X_1,X_2})$, where $\mathcal{X}_k$ is the input alphabet of User k, $k = 1, 2$, $\mathcal{Y}$ is the output alphabet, and $P_{Y|X_1,X_2}$ is the transition probability matrix from $\mathcal{X}_1 \times \mathcal{X}_2$ to $\mathcal{Y}$. The channel is memoryless without feedback.

In this section we present achievable rates for the MAC with an unreliable cribbing - that may or may not be present - from Encoder 1 to Encoder 2. The basic assumptions are as follows. Since Encoder 2 listens to Encoder 1, he knows whether the cribbing link is present. Similarly, the decoder knows it since Encoder 2 can convey to him this message, as it is only one bit of information to transmit. Encoder 1, on the other hand, does not know whether the cribbing link is present, since he cannot be informed about it. He is only aware that cribbing could occur. Let $\mathcal{N}_1' = \{1, 2, \dots, \nu_1'\}$ and $\mathcal{N}_2'' = \{1, 2, \dots, \nu_2''\}$ be two message sets. A coding scheme operates as follows. Four messages M_1, M_1', M_2, and M_2'' are drawn uniformly and independently from the sets $\mathcal{N}_1$, $\mathcal{N}_1'$, $\mathcal{N}_2$, $\mathcal{N}_2''$, respectively. Encoder 1 maps the pair (M_1, M_1') to an input sequence $\boldsymbol{x}_1 = \boldsymbol{x}_1(M_1, M_1')$. If the cribbing link is absent, Encoder 2 maps the message M_2 to to an input sequence $\boldsymbol{x}_2 = \boldsymbol{x}_2(M_2)$. If the cribbing link is present, Encoder 2 knows $\boldsymbol{x}_1$ causally, thus maps the pair $(M_2'', \boldsymbol{x}_1)$ to an input sequence $\boldsymbol{x_2}$, in a causal manner:

$$\boldsymbol{x}_2(m_2'', \boldsymbol{x}_1) = (x_{2,1}(m_2'', x_{1,1}), x_{2,2}(m_2'', x_1^2), \dots, x_{2,n}(m_2'', x_1^n)) \quad (7)$$

At the output, the decoder decodes (M_1, M_2) if cribbing is absent, and (M_1, M_1', M_2'') if cribbing is present.

Remark 3: Note that there is a slight difference in the interpretation of the message sets, compared to the message sets of the BC model studied in Section II. The pair (M_1, M_1') is encoded by User 1, where M_1 is always decoded, and M_1' is decoded only if cribbing is present. For User 2, if cribbing is absent, M_2 is encoded, whereas if cribbing is present, M_2'' is encoded. Therefore User 2 can reduce his rate in case of cribbing, in favor of increasing the rate of User 1. Due to this structure, the joint distribution of M_2 and M_2'' is immaterial, as they never appear together in the coding scheme.

Following is a formal definition of the scheme described above.

Definition 2: An $(n, \nu_1, \nu_1', \nu_2, \nu_2'', \epsilon)$ code for the MAC $P_{Y|X_1,X_2}$ with unreliable cribbing link consist of $n + 2$ encoding maps

$$f_1 : \mathcal{N}_1 \times \mathcal{N}_1' \to \mathcal{X}_1^n \quad (8a)$$

$$f_2 : \mathcal{N}_2 \to \mathcal{X}_2^n \quad (8b)$$

$$f_{2,i}'' : \mathcal{N}_2'' \times \mathcal{X}_1^i \to \mathcal{X}_{2,i}, \quad i = 1, 2, \dots, n \quad (8c)$$

and a pair of decoding maps

$$g : \mathcal{Y}^n \to \mathcal{N}_1 \times \mathcal{N}_2 \quad (9a)$$

$$g' : \mathcal{Y}^n \to \mathcal{N}_1 \times \mathcal{N}_1' \times \mathcal{N}_2'' \quad (9b)$$

such that the average probabilities of error P_e and P_e' do not exceed ϵ. Here

$$P_e = \frac{1}{\nu_1 \nu_1' \nu_2} \sum_{m_1, m_1', m_2} P_{Y|X_1,X_2}(\mathcal{Q}_e | f_1(m_1, m_1'), f_2(m_2)) \quad (10a)$$

$$P_e' = \frac{1}{\nu_1 \nu_1' \nu_2''} \sum_{m_1, m_1', m_2''} P_{Y|X_1,X_2}(\mathcal{Q}_e' | f_1(m_1, m_1'), \boldsymbol{f}_2''(m_2'', f_1(m_1, m_1'))) \quad (10b)$$

where $\boldsymbol{f}_2''(m_2'', f_1(m_1, m_1'))$ is the sequence of maps $f_{2,i}''$ in (8c), the sets $\mathcal{Q}_e$ and $\mathcal{Q}_e'$ are defined as

$$\mathcal{Q}_e(m_1, m_2) = \{\boldsymbol{y} : g(\boldsymbol{y}) \neq (m_1, m_2)\}$$
$$\mathcal{Q}_e'(m_1, m_1', m_2'') = \{\boldsymbol{y} : g'(\boldsymbol{y}) \neq (m_1, m_1', m_2'')\}$$

and the dependence of the sets $\mathcal{Q}_e$, $\mathcal{Q}_e'$ on the messages is dropped in (10), for notational convenience.

The rates (R_1, R_1', R_2, R_2''), and achievability of a given quadruple, are defined as usual. The capacity region of the MAC with unreliable cribbing is the closure of the collection of all achievable quadruples (R_1, R_1', R_2, R_2''), and is denoted by $\mathcal{C}_{\text{mac}}$. Our interest is in characterizing $\mathcal{C}_{\text{mac}}$.

Let $\mathcal{V}$ be a finite set, and let $\mathcal{P}$ be the collection of all joint distributions $P_{V,X_1,X_2,X_2'',Y,Y''}$ of the form

$$P_{V,X_1} P_{X_2} P_{Y|X_1,X_2} P_{X_2''|X_1,V} P_{Y''|X_1,X_2''}, \quad (11)$$

where $P_{Y''|X_1,X_2''}$ is our MAC with X_2'' at the input of Encoder 2. The interpretation of this joint distribution is as follows. The pair (V, X_1) are the coding random variables of User 1. These are fixed, regardless of whether cribbing is present or not. The input X_2 is the coding variable of User 2 if cribbing is absent, therefore it is independent of (V, X_1), and Y is the MAC output due to inputs X_1, X_2. When cribbing is present, User 2 encodes with X_2'' which can depend on V and X_1. The output of the channel due to inputs X_1 and X_2'' is denoted by Y''.

Let $\mathcal{I}_{\text{mac}}$ be the collection of all quadruples (R_1, R_1', R_2, R_2'') satisfying

$$R_1 \leq I(V; Y|X_2) \quad (12a)$$

$$R_2 \leq I(X_2; Y|V) \quad (12b)$$

$$R_1 + R_2 \leq I(V, X_2; Y) \quad (12c)$$

$$R_1' \leq H(X_1|V) \quad (12d)$$

$$R_2'' \leq I(X_2''; Y''|X_1, V) \quad (12e)$$

$$R_1 + R_1' + R_2'' \leq I(X_1, X_2''; Y'') \quad (12f)$$

for some $P_{V,X_1,X_2,X_2'',Y,Y''} \in \mathcal{P}$. We have

Theorem 2: For any MAC with unreliable cribbing

$$\mathcal{I}_{\text{mac}} \subseteq \mathcal{C}_{\text{mac}}.$$

The proof of Theorem 2 is based on the combination of superposition coding and block-Markov coding. The transmission is always performed in B sub-blocks, of length n each. In each sub-block, the messages of User 1 are encoded in two layers. First the message M_1 is encoded with V, and then the message M_1' is encoded with X_1, using superposition coding around the cloud centers V. If the cribbing link is absent, Encoder 2 encodes his messages independently of Encoder 1. The decoder can then decode only the messages of V and X_2. If the cribbing link is present, block Markov coding is employed, similarly to the scheme used in [9] for one sided causal cribbing. Due to lack of space, the details are omitted.

Note that when cribbing is absent, the rates R_1' and R_2'' are not decoded. Thus, setting $V = X_1$ in the region $\mathcal{I}_{\text{mac}}$ yields the capacity region of the MAC without cribbing.

IV. Proofs

In this section a sketch of the proof of Theorem 1 is given. The direct part uses random selection and strong typicality arguments. The definitions and notation follow closely the convention in [1], and therefore are omitted.

Direct Part. We use the binning approach suggested in [4].

Code construction. Fix a joint distribution $P_{U,V,X}$.

1) Generate 2^{nR_2} codewords $\boldsymbol{u}(j)$, $j = 1, 2, \dots 2^{nR_2}$, i.i.d., according to P_U.
2) For every $\boldsymbol{u}(j)$, generate $2^{nR_2'}$ codewords $\boldsymbol{v}(k|j)$, $k = 1, 2, \dots 2^{nR_2'}$, independently according to $\prod_{i=1}^n P_{V|U}(v_i|u_i(j))$.
3) For every j, distribute the $2^{nR_2'}$ codewords $\boldsymbol{v}(k|j)$, $k = 1, 2, \dots, 2^{nR_2'}$, into $2^{nC_{1,2}}$ bins, evenly and independently of each other. Thus, in every bin there are $2^{n(R_2'-C_{1,2})}$ codewords $\boldsymbol{v}(k|j)$ with a fixed index j. Denote by $b(k|j)$ the bin number to which $\boldsymbol{v}(k|j)$ belongs. Note that
$$1 \leq b(k|j) \leq 2^{nC_{1,2}}. \tag{13}$$
4) For every pair $(\boldsymbol{u}(j), \boldsymbol{v}(k|j))$, $j = 1, 2, \dots, 2^{nR_2}$, $k = 1, 2, \dots, 2^{nR_2'}$, generate 2^{nR_1} vectors $\boldsymbol{x}(l|j,k)$, $l = 1, 2, \dots, 2^{nR_1}$, independently of each other, according to $\prod_{i=1}^n P_{X|U,V}(x_i|u_i(j), v_i(k|j))$.

Encoding. Given a triple (j,k,l), where $j = 1, 2, \dots, 2^{nR_2}$, $k = 1, 2, \dots, 2^{nR_2'}$, $l = 1, 2, \dots, 2^{nR_1}$, the encoder sends via the channel the codeword $\boldsymbol{x}(l|j,k)$.

Decoding. We assume first that the conference link is absent. Decoder 2 has y_2^n at hand. He looks for the unique index $\hat{j}$ in $\{1, 2, \dots, 2^{nR_2}\}$ such that
$$(\boldsymbol{u}(\hat{j}), y_2^n) \in T_{U,Y_2}$$
If such $\hat{j}$ does not exist, or there is more than one such index, an error is declared. By classical results, if
$$R_2 < I(U;Y_2) \tag{14}$$
the index j is decoded correctly with high probability.

Decoder 1 has y_1^n at hand. He looks for the unique index $\hat{\hat{j}}$ in $\{1, 2, \dots, 2^{nR_2}\}$ such that
$$(\boldsymbol{u}(\hat{\hat{j}}), y_1^n) \in T_{U,Y_1}$$
If such $\hat{\hat{j}}$ does not exist, or there is more than one such index, an error is declared. By classical results, if
$$R_2 < I(U;Y_1) \tag{15}$$
Decoder 1 succeeds to decode correctly the index j with high probability. Since the channel is degraded, if (14) holds, it implies (15). Next, Decoder 1 looks for the uniques index $\hat{\hat{k}}$ in $\{1, 2, \dots, 2^{nR_2'}\}$ such that
$$(\boldsymbol{u}(\hat{\hat{j}}), \boldsymbol{v}(\hat{\hat{k}}|\hat{\hat{j}}), y_1^n) \in T_{VY_1|U}(\boldsymbol{u}(\hat{\hat{j}})) \tag{16}$$
If such $\hat{\hat{k}}$ does not exist, or there is more than one such, an error is declared. By classical results, the index $k \in \{1, 2, \dots, 2^{nR_2'}\}$ is decoded correctly with high probability if
$$R_2' < I(V;Y_1|U) \tag{17}$$
Having the pair $(\hat{\hat{j}}, \hat{\hat{k}})$ at hand, Decoder 1 looks for the unique index $\hat{\hat{l}} \in \{1, 2, \dots, 2^{nR_1}\}$ satisfying
$$(\boldsymbol{u}(\hat{\hat{j}}), \boldsymbol{v}(\hat{\hat{k}}|\hat{\hat{j}}), \boldsymbol{x}(\hat{\hat{l}}|\hat{\hat{k}}, \hat{\hat{j}}), y_1^n) \in T_{XY_1|U,V}(\boldsymbol{u}(\hat{\hat{j}}, \boldsymbol{v}(\hat{\hat{k}}|\hat{\hat{j}})) \tag{18}$$
By classical results, this step succeeds if the rate R_1 satisfies
$$R_1 < I(X;Y_1|U,V). \tag{19}$$
This concludes the decoding process when the conference link is absent. By (14), (17) and (19), the conditions for correct decoding when there is no conferencing are
$$R_2 \leq I(U;Y_2) \tag{20a}$$
$$R_2' \leq I(V;Y_1|U) \tag{20b}$$
$$R_1 \leq I(X;Y_1|U,V) \tag{20c}$$
Observe that, although the rate R_2' is decoded by Decoder 1 (if (20b) is satisfied), it does not arrive to User 2, since the conferencing link is absent. The bound (20b) is still needed in order to guarantee that Decoder 1 can proceed and decode the index l (the message intended to him).

We turn now to the case where the conference link is present. Decoder 1 operates exactly as in the case of no conference, and decodes the indices $\hat{\hat{j}}$, $\hat{\hat{k}}$, and $\hat{\hat{l}}$. If (20) hold, these steps succeed with high probability. He then sends $b(\hat{\hat{k}}|\hat{\hat{j}})$, the index of the bin to which $\boldsymbol{v}(\hat{\hat{k}}|\hat{\hat{j}})$ belongs, via the conference link. Due to (13), the link capacity suffices, and Decoder 2 receives $b(\hat{\hat{k}}|\hat{\hat{j}})$ without an error.

Decoder 2 decodes the index $\hat{j}$ as in the case of no conference. After receiving from Decoder 1 the bin index $b(\hat{\hat{k}}|\hat{\hat{j}})$, he looks in this bin for the unique index $\hat{k}$ such that
$$(\boldsymbol{v}(\hat{k}|\hat{j}), \boldsymbol{u}(\hat{j}), \boldsymbol{y}_2) \in T_{V,Y_2|U}(\boldsymbol{u}(\hat{j}). \tag{21}$$

If such an index does not exist, or there is more than one such, an error is declared. From the code construction, every bin contains approximately $2^{n(R_2'-C_{1,2})}$ codewords $\boldsymbol{v}$. Assuming that the previous decoding steps were successful (i.e., $\hat{\hat{j}}$, $\hat{\hat{k}}$, $\hat{j}$ are the correct indices j, k, and j, respectively), by classical results $\hat{k}$ is correct with high probability if

$$R_2' - C_{1,2} \leq I(V;Y_2|U) \tag{22}$$

The region defined by (20) and (22) coincides with $\mathcal{R}(C_{1,2})$. This concludes the proof of the achievability part.

Converse Part. We start with a sequence of codes $(n, 2^{nR_1}, 2^{nR_2}, 2^{nR_2'}, 2^{nC_{1,2}}, \epsilon_n)$ with increasing blocklength n, satisfying $\lim_{n\to\infty} \epsilon_n = 0$. We denote by M_k the random message from $\mathcal{N}_k$, $k=1,2$, and by M_2' the message from $\mathcal{N}_2'$. The conference message is denoted by $M_{1,2}$. By Fano's inequality we can bound the rate R_2 as

$$\begin{aligned} nR_2 - n\delta_n \leq I(M_2;Y_2^n) &= \sum_{i=1}^n I(M_2;Y_{2,i}|Y_2^{i-1}) \\ &\leq \sum_{i=1}^n I(M_2,Y_2^{i-1};Y_{2,i}) \end{aligned} \tag{23}$$

where $\lim_{n\to\infty}\delta_n = 0$, due to $\lim_{n\to\infty}\epsilon_n = 0$. We now bound the rate R_2' as follows. If the conference link is present, then the messages M_2' can be decoded by Decoder 2 based on Y_2^n and the message transmitted via the conference link, $M_{1,2}$. Therefore

$$\begin{aligned} nR_2' - n\delta_n &\leq I(M_2';Y_2^n,M_{1,2}|M_2) \qquad (24) \\ &= I(M_2';Y_2^n|M_2) + I(M_2';M_{1,2}|M_2,Y_2^n) \\ &\leq I(M_2';Y_2^n|M_2) + H(M_{1,2}) \\ &= \sum_{i=1}^n I(M_2';Y_{2,i}|M_2,Y_2^{i-1}) + H(M_{1,2}) \\ &\leq \sum_{i=1}^n I(M_2',Y_1^{i-1};Y_{2,i}|M_2,Y_2^{i-1}) + H(M_{1,2}) \end{aligned}$$

Moreover, the message M_2' can be decoded by Decoder 1, regardless of the conference link. Hence:

$$\begin{aligned} nR_2' - n\delta_n &\leq I(M_2';Y_1^n|M_2) \\ &= \sum_{i=1}^n I(M_2';Y_{1,i}|M_2,Y_1^{i-1}) \\ &\overset{(a)}{=} \sum_{i=1}^n I(M_2';Y_{1,i}|M_2,Y_1^{i-1},Y_2^{i-1}) \\ &\leq \sum_{i=1}^n I(M_2',Y_1^{i-1};Y_{1,i}|M_2,Y_2^{i-1}) \end{aligned} \tag{25}$$

where (a) is true because the channel is physically degraded.

The rate R_1 can be bounded by

$$\begin{aligned} nR_1 - n\delta_n &\leq I(M_1;Y_1^n|M_2,M_2') \\ &= \sum_{i=1}^n I(M_1;Y_{1,i}|M_2,M_2',Y_1^{i-1}) \\ &\overset{(a)}{=} \sum_{i=1}^n I(M_1;Y_{1,i}|M_2,M_2',Y_1^{i-1},Y_2^{i-1}) \\ &\overset{(b)}{=} \sum_{i=1}^n I(X_i;Y_{1,i}|M_2,M_2',Y_1^{i-1},Y_2^{i-1}) \end{aligned} \tag{26}$$

where (a) is true since the channel is physically degraded. Equality (b) holds since X_i is a deterministic function of the messages M_1, M_2, and M_2', and since $Y_{1,i}$ is independent of $(M_2,M_2',Y_2^{i-1},Y_1^{i-1},M_1)$ when conditioned on X_i. Defining $U_i=(M_2,Y_2^{i-1})$, $V_i=(M_2',Y_1^{i-1})$ and using the fact that

$$\frac{1}{n}H(M_{1,2}) \leq C_{1,2}, \tag{27}$$

we obtain from (23), (24), (25), and (26) the bounds

$$R_2 - \delta_n \leq \frac{1}{n}\sum_{i=1}^n I(U_i;Y_{2,i}) \tag{28a}$$

$$R_2' - \delta_n \leq \frac{1}{n}\sum_{i=1}^n I(V_i;Y_{2,i}|U_i) + C_{1,2} \tag{28b}$$

$$R_2' - \delta_n \leq \sum_{i=1}^n I(U_i;Y_{1,i}|V_i) \tag{28c}$$

$$R_1 - \delta_n \leq \sum_{i=1}^n I(X_i;Y_{1,i}|U_i,V_i) \tag{28d}$$

The converse part now follows by using the convexity of $\mathcal{R}(C_{1,2})$ and taking the limit of large n in (28). □

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